CAHSEE ON TARGET
SCHOOL/UNIVERSITY PARTNERSHIPS, UC DAVIS

ANSWER KEY
STATISTICS AND
PROBABILITIES
CAHSEE on Target
UC Davis, School and University Partnerships

CAHSEE ON TARGET
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Introduction to the CAHSEE

The CAHSEE stands for the California High School Exit Exam. The mathematics section of the CAHSEE consists of 80 multiple-choice questions that cover 53 standards across 6 strands. These strands include the following:

Number Sense (14 Questions)

Statistics, Data Analysis & Probability (12 Questions)

Algebra & Functions (17 Questions)

Measurement & Geometry (17 Questions)

Mathematical Reasoning (8 Questions)

Algebra 1 (12 Questions)

What is CAHSEE on Target?

CAHSEE on Target is a tutoring course specifically designed for the California High School Exit Exam (CAHSEE). The goal of the program is to pinpoint each student’s areas of weakness and to then address those weaknesses through classroom and small group instruction, concentrated review, computer tutorials and challenging games.

Each student will receive a separate workbook for each strand and will use these workbooks during their tutoring sessions. These workbooks will present and explain each concept covered on the CAHSEE, and introduce new or alternative approaches to solving math problems.

What is Statistics & Probability?

Statistics is the collection and classification of data (information). Data can be represented in many forms, including pie charts, line graphs, bar graphs, and scatter plots. Units 1 and 2 in this workbook deal with statistics. Probability examines the possible outcomes of events and the likelihood that any one event will occur. The probability of an event is generally defined as a number between 0 (impossibility) and 1 (absolute certainty). Unit 3 deals with probability.
A data set is a group of numbers. The mean, median, and mode of a data set are statistical measures that give information about the "averages" of those numbers.

Let's look at each of these measures separately:

A. Mean

The mean is the most common measure of average.

To find the mean of a group of numbers, add all of the values in the data set and then divide by the number of values in the data set.

Example: Find the mean of this data set: \{6, 9, 7, 5, 4, 9, 9\}

Steps:

- Add all of the values in the data set:

\[
6 + 9 + 7 + 5 + 4 + 9 + 9 = 49
\]

- Divide the sum by the number of values in the data set:

\[
\frac{\text{Sum}}{\text{Number of Values}} = \frac{49}{7}
\]

The mean is 7.
The mean can also be a decimal. Look at the next example: $6, 1.1$

**Example:** The chart below shows the scores for five students on last week's trigonometry quiz. (Note: The highest possible score was 10.) Find the mean score.

<table>
<thead>
<tr>
<th>Student</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melba</td>
<td>9</td>
</tr>
<tr>
<td>Christina</td>
<td>8</td>
</tr>
<tr>
<td>Danielle</td>
<td>9</td>
</tr>
<tr>
<td>Jarell</td>
<td>8</td>
</tr>
<tr>
<td>Monique</td>
<td>8</td>
</tr>
</tbody>
</table>

**Steps:**

- Add all of the values in the data set:

$$9 + 8 + 9 + 8 + 8 = 42$$

- Divide the sum by the number of values in the data set:

$$\text{Sum} \rightarrow 42$$
$$\text{Number of Values} \rightarrow 5$$

**The mean score is** $8 \frac{2}{5}$, which is equal to 8.4

Notice that the mean score is **not an actual value in the data set**.
1. Find the mean of the data set \{3, 2, 5, 7, 3, 8, 7\}

Answer: \( \frac{35}{7} = 5 \)

2. Find the mean of the data set \{6, 8, 11, 10, 8, 7, 13\}

Answer: \( \frac{63}{7} = 9 \)

3. Find the mean of the data set \{9, 4, 6, 8, 9, 5, 8, 7\}

Answer: \( \frac{56}{8} = 7 \)
4. The chart below shows the number of ice cream cones sold at Buddy's Ice Cream Parlor from Monday through Friday:

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla</td>
<td>15</td>
<td>19</td>
<td>16</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Chocolate</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Strawberry</td>
<td>12</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

What was the mean sale for vanilla?

**Answer:** \[ \frac{85}{5} = 17 \]

What was the mean sale for all flavors on Monday?

**Answer:** \[ \frac{49}{3} = 16 \frac{1}{3} = 16.33 \]
The median is the **middle** value in a data set.

The steps for finding the median depend on whether the number of values in the data set is **odd** or **even**. It is always easier to find the median if you have an **odd** number of values.

### i. Odd Number of Values

When the number of values in the data set is **odd**, the **median** is the **middle** value; it has one-half of the values on either side. To find the median, **order** the values from **smallest to largest** and find the **middle** value.

**Example:** Find the median of the data set: \{6, 9, 7, 5, 4, 9, 9\}

**Steps:**

- **Order** the values in the data set from smallest to largest:

  \{4, 5, 6, 7, 9, 9, 9\}

- Determine whether the number of values in your data set is **odd** or **even**:

  There are 7 values in the data set. 7 is an **odd** number.

- Apply the rule for an odd number of values: Find the **middle** value.

  The **median** is 7.
On Your Own

1. Find the median of the data set: {3, 2, 5, 7, 3, 8, 7}.

    \[\begin{array}{cccccc}
    2 & 3 & 3 & 5 & 7 & 7 & 8 \\
    \end{array}\]

    Answer: The median is 5.

2. In 2004, seven homes were sold in the city of Fairview. The selling prices were as follows:

    \$221,500 \hspace{0.5cm} \$258,400 \hspace{0.5cm} \$237,800 \hspace{0.5cm} \$274,000 \hspace{0.5cm} \$232,700

    Find the median price.

    Answer: $237,800

3. Mr. Martin had seven students in his after-school algebra tutorial. The scores they received on their last quiz were as follows:

    \[\begin{array}{ccccccc}
    81 & 73 & 84 & 78 & 89 & 82 & 81 \\
    \end{array}\]

    What was the median score?

    Answer: 81
4. The chart below shows the number of ice cream cones sold at Buddy's Ice Cream Parlor from Monday through Friday:

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
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<tbody>
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<td>15</td>
<td>19</td>
<td>16</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Chocolate</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Strawberry</td>
<td>12</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

Find the median daily sales for chocolate.

**Answer: 22**
ii. **Even Number of Values**

If the number of values in the data set is **even**, order the values from **smallest to largest** and then take the **mean of the two middle values**.

**Example:** Find the median of the data set: \( \{6, 9, 7, 5, 4, 9, 9, 5\} \)

**Steps:**

- **Order** the values in the data set from smallest to largest:

  \[ \{4, 5, 5, 6, 7, 9, 9, 9\} \]

- Determine whether the number of values in your data set is **odd** or **even**:

  There are 8 values in the data set. 8 is an **even** number.

- Apply the rule for an even number of values: Find the mean of the two **middle** values:

  \[ \{4, 5, 5, \underline{6}, \underline{7}, 9, 9, 9\} \]

  The two middle values are 6 and 7.

  The mean of 6 and 7 is found by adding the two numbers and then dividing by 2:

  \[ \frac{6 + 7}{2} = \frac{13}{2} = 6.5 \]

  The **median** is 6.5.
On Your Own

1. Find the median of the data set: \{11, 15, 11, 9, 13, 12, 12, 14\}

\[
\begin{array}{c}
9 & 11 & 11 & 12 & 12 & 13 & 14 & 15 \\
\end{array}
\]

Answer: 12

2. Find the median of the data set: \{19, 21, 22, 20, 18, 21, 23, 18\}

\[
\begin{array}{c}
18 & 18 & 19 & 20 & 21 & 21 & 22 & 23 \\
\end{array}
\]

Answer: 20.5

3. Find the median of the data set: \{35, 29, 33, 38, 31, 34\}

\[
\begin{array}{c}
29 & 31 & 33 & 34 & 35 & 38 \\
\end{array}
\]

Answer: 33.5
C. Mode

The mode is the value that occurs most often in the set.

To find the mode of a data set, order the values from smallest to largest and find the value that occurs most often.

Example: Find the mode of this data set: \{6, 9, 7, 5, 4, 9, 9\}

Steps:

• Order the values from smallest to largest:

\{4, 5, 6, 7, 9, 9, 9\}

• Determine which value appears most often.

\{4, 5, 6, 7, 9, 9, 9\}

The value 9 occurs 3 times. 9 is the mode.

Two or More Modes

It is possible to have more than one mode.

Example: Find the mode of this data set: \{6, 7, 5, 6, 4, 9, 9\}

• Order the values from smallest to largest:

\{4, 5, 6, 6, 7, 9, 9\}

• Determine which value appears most often

\{4, 5, 6, 6, 7, 9, 9\}

There are 2 modes: 6 and 9. They both appear twice.
No Mode

It is possible to have a data set with no mode.

Example: Find the mode of this data set: {6, 9, 7, 5, 4, 3, 2}

Steps:
- Order the values from smallest to largest:
  \{2, 3, 4, 5, 6, 7, 9\}
- Determine which value appears most often.
  Each value appears only once. There is no mode.

On Your Own

1. Find the mode of this data set: \{3, 2, 5, 7, 3, 8, 7\}
   \textbf{Answer: 3 & 7}

2. Find the mode of this data set: \{1, 2, 4, 5, 4, 3, 2, 8, 6\}
   \textbf{Answer: 2 & 4}

3. Find the mode of this data set: \{7, 3, 7, 3, 5, 7, 4\}
   \textbf{Answer: 7}

4. Find the mode of this data set: \{8, 1, 4, 3, 2, 5, 6, 7\}
   \textbf{Answer: No mode}

5. Find the mode of this data set: \{5, 9, 12, 3, 8, 5, 7, 6\}
   \textbf{Answer: 5}
Tricks to Remember Mean, Median & Mode

Use the following tricks to remember these terms:

- **Mean**: Even It Out (Add & Divide)
- **Median**: Middle Value
- **Mode**: Most Often

**Practice**

1. Find the mean and median of the data set: \{54, 33, 28, 40, 52\}

   - **Mean**: 41.4
   - **Median**: 40

2. Find the median and mode of the data set: \{13, 12, 12, 11, 11, 11\}

   - **Median**: 11.5
   - **Mode**: 11
3. Find the mean, median and mode of the data set:

\{21, 23, 23, 19, 20, 21, 22\}

Mean: 21.29
Median: 21
Mode: 21 & 23

4. The chart below shows the number of ice cream cones sold at Buddy's Ice Cream Parlor from Monday through Friday:

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla</td>
<td>15</td>
<td>19</td>
<td>16</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Chocolate</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Strawberry</td>
<td>12</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

What was the mode for vanilla ice cream? **No mode**
What was the mode for chocolate ice cream? **22**
What was the mode for strawberry ice cream? **14**
What was the mode for Thursday sales? **18**
What was the mode for Wednesday sales? **No mode**
5. The chart below shows the science test scores for three students at Brown High School. (Note: The highest possible score for each test is 20.)

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erica</td>
<td>17</td>
<td>18</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Briana</td>
<td>17</td>
<td>16</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>Damiana</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

a. Who had the highest mean score? What was it?

**Damiana had the highest mean score, which was 18.**

b. What was the median score for Test 2?

16 18 18

18

c. What was the mean score for Test 3?

\[
\frac{54}{3} = 18
\]
Unit Quiz: The following questions appeared on the CAHSEE.

1. Rico’s first three test scores in biology were 65, 90, and 73. What was his mean score?
   A. 65
   B. 73
   C. 76
   D. 90
   Standard: Grade 6, 1.1

2. The chart below shows the mathematics test scores of three students.

<table>
<thead>
<tr>
<th>Student</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parissa</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Hector</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Charles</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

   What is Hector’s mean score?
   A. 6
   B. 7
   C. 8
   D. 9
   Standard: Grade 6, 1.1

3. Donald priced six personal Compact Disc (CD) players. The prices are shown below:

   $21.00, $23.00, $21.00, $39.00, $25.00, $31.00

   What is the median price?
   A. $21.00
   B. $24.00
   C. $27.00
   D. $30.00
   Standard: Grade 6, 1.1
Unit 2: Statistics: Representing & Interpreting Data

Data is information. Data is often presented visually, as in tables, graphs and charts; this allows us to easily compare the data and draw conclusions about what it means.

A. Tables

Tables organize information using columns (vertically) and rows (horizontally).

The following chart shows the mean grade given by each of the five math teachers at Montgomery High School:

Mean Grade Given by Teachers at Montgomery High School

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Grade Taught</th>
<th>Subject</th>
<th>Mean Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Brody</td>
<td>9th</td>
<td>Algebra I</td>
<td>B+</td>
</tr>
<tr>
<td>Mr. Harvey</td>
<td>10th</td>
<td>Geometry</td>
<td>B-</td>
</tr>
<tr>
<td>Mr. Purnell</td>
<td>11th</td>
<td>Algebra II</td>
<td>B</td>
</tr>
<tr>
<td>Ms. Anderson</td>
<td>12th</td>
<td>Calculus</td>
<td>A</td>
</tr>
<tr>
<td>Mr. Harris</td>
<td>12th</td>
<td>Trigonometry</td>
<td>A-</td>
</tr>
</tbody>
</table>

The title tells us what the table is about.

The column headings tell us what data is in each column:

- the name of the teacher
- the grade taught
- the subject taught
- the mean grade given to students

You can use the table to find out information about each teacher by looking at each row. For example, Ms. Brody teaches Algebra I to 9th graders; the mean grade given to students in her class is B+.

What was the mean grade given by Ms. Anderson? A

Who teaches Algebra II? Mr. Purnell
Graphs are diagrams that show how two or more variables (or sets of data) are related to one another. Graphs are drawn on a grid with two axes: a horizontal axis and a vertical axis.

On the CAHSEE, you will need to read and interpret bar graphs, line graphs, and pie charts (or circle graphs). In all three cases, be sure to pay attention to three things:

- the title
- the scale
- the axis headings

i. Bar Graphs

The main purpose of bar graphs is to compare quantities. Bar graphs can be horizontal or vertical. One advantage of bar graphs is that they show distinct categories and provide an easy way to compare them; they can also be used to show single categories at specific intervals of time.

Example: The bar graph below represents the results of a class survey in which students were asked to vote for their favorite color.

![Survey Results for Favorite Colors](image)
Example: The following bar graph shows the average distance (in yards) that students ran each week in gym class.

The title of the graph is "Average Distance Covered by Students."

The x-axis represents the weeks in which the students were in class.

The y-axis represents the average distance (as measured in yards) covered by the students in the class.

What is the scale of the y-axis? In other words, how many yards does one horizontal line represent? 25 yards

What conclusions, if any, can you draw from the graph?

Students made great progress with each week of practice. The more you spend time running, the farther you can run.

What was the average distance for Week 3? 250 yards
1. The vertical bar graph represents the average rainfall (as measured in inches) in Morristown from January through June.

What is the title of the graph? **Average Monthly Rainfall in Morristown**
What is measured on the y-axis? **Inches of Rainfall**
What is the scale on the y-axis? **Each line represents 5 inches**
Which month had the most rainfall? **June**

The month of June receives, on average, more rainfall than which of the two months combined?

A. **January and February**
B. February and March
C. March and April
D. April and May
2. The bar graph below shows the number of hundreds of homes sold in Rady County from January through May.

How many homes were sold in March? **200**

How many more homes were sold in April than in May? **300**

How many fewer homes were sold in March than in February? **400**
3. The bar graph shows the production of frozen desserts in February 2003.

Source: USDA-NASS (National Agricultural Statistics Service)

Approximately how many gallons of regular \textbf{and} low-fat ice cream were produced in February 2003? \textbf{80 million}
ii. Double Bar Graphs \hspace{1cm} 7, 1.1 & 6, 2.5

A double bar graph shows two sets of data side by side. It is often used to compare the results of two or more data sets.

On the CAHSEE, you may be asked to interpret a double bar graph.

Example: The double bar graph below shows the number of students, in grades 9 through 12, who received a score of B+ or higher for both math and English.

What conclusions can be drawn from the graph?

A. **On average, more students received a B+ or higher in math than in English.**

B. On average, more students received a B+ or higher in English than in math.

C. The number of students receiving a B+ or higher is the same for both math and English.

D. As each year progresses, more students receive a B+ or higher in both math and English.
iii. Line Graphs

Line graphs show **change over time** and how **one variable** is affected by a **change in another variable**.

**Example:** The line graph below shows the frequency of phone calls made from Monday through Sunday in the town of Hopscotch.

![Phone Calls Made in the Town of Hopscotch](image)

The **title** of the graph is "Phone Calls Made in the Town of Hopscotch"
The **y-axis** shows the **number** of phone calls (in thousands) made each day.

The **x-axis** shows the **days** of the week.

On which **day** were the most calls made? **Saturday**

How many more phone calls were made on Saturday than Sunday? **3,000**
Identifying Trends & Predicting the Future with Line Graphs

One advantage of line graphs is that they allow one to **identify trends** and **predict** what will occur in the **future**.

**Example:** The line graph below represents the annual population (in thousands of people) of Elliot City from 2001 to 2005.

Which year had the greatest increase in population over that of the previous year?

A. 2001  
**B. 2002**  
C. 2003  
D. 2004  
E. 2005

Which of the following is the most likely figure for the population of Elliot City in 2006?

A. 52,000  
B. 62,000  
**C. 72,000**  
D. 82,000
1. The line graph shows the changes in the federal hourly minimum wage from October 1938 to September 1997.

The largest increase in the minimum wage (as measured in cents) occurred between

A. January 1981 and April 1990
B. April 1990 and April 1991
C. April 1991 and October 1996
D. October 1996 and September 1997
2. The line graph below shows the number of claims filed with the Lost & Found from January through October.

How many fewer claims were filed in February than in January? \(400\)

Which of the following is the most probable explanation for the marked decrease in claims filed during the summer months?

A. People lose fewer items in the summer because they are more careful then.
B. People lose fewer items during the winter because they are more careful then.
C. **During the winter months, people wear more items, such as umbrellas, gloves and scarves, that they could lose.**
D. People place greater value on the items they use during the summer months than they do on the items they use during the winter months.
3. The line graph below shows the daily temperature (measured in degrees) from Monday through Sunday.

Between which two days did the largest change in temperature occur?

A. Monday and Tuesday
B. Tuesday and Wednesday
C. Wednesday and Thursday
D. Thursday and Friday
E. Friday and Saturday
F. Saturday and Sunday

What was the approximate temperature on Friday? 22.5º
4. The following line graph shows the distribution of grades in Ms. Grey's Art Class.

How many students received an A or a B? 14

How many more students received a B than a C? 3
iv. **Double Line Graphs**  

On the CAHSEE, you may be asked to interpret a **double line graph**.

A double line graph shows **two sets of data side by side**. It is often used to **compare** the results and trends of two or more data sets.

**Example:** The following line graph shows the expenses for food and telephone incurred by Maria from January through June.

![Double Line Graph](image)

How much did Maria spend on both food and the telephone in March?  **$20**

In which month did Maria spend the same amount of money for both food and phone use? **February**

How much more did Maria spend on food than the telephone in June?  **$35**

In which months did the phone exceed that of the food bill?  **January & April**
Pie charts (also called circle graphs) are circular, like a pie, and compare the relationship of the parts to the whole. Each section of the pie represents a percent or fraction of the whole, which is 1 (or 100%).

Example: The pie graph below shows the age distribution of those who receive a Graduate Equivalency Diploma (GED).

Notice that the distribution is shown in terms of percent. The circle is divided into sections; each section represents a different category.

In the above pie graph, there are five distinct age categories:

- 19 and under
- 20 - 24
- 25 - 29
- 30 - 34
- 35 +

One advantage of pie charts is that the reader can see right away which section is the biggest and which is the smallest.

In the above graph, the greatest proportion of GED recipients is the 19 and under category.
On Your Own

1. The pie chart shows cheese production, by percent, in 2003.

Source: USDA-NASS (National Agricultural Statistics Service)

According to the pie chart above, which two types of cheese accounted for more than forty percent of all production in 2003?

A. Mozzarella and Swiss

B. Cheddar and Other Italian

C. Other American and All Other

D. Other American and Other Italian

Tutors: Point out to students that, while Cheddar & Mozzarella also accounted for more than 40% of all production, this option did not appear as one of the answer choices.
2. The following graph shows the value of U.S. crops sold in 1997.

![Value of Crops Sold: 1997](image)

Source: USDA-NASS (National Agricultural Statistics Service)

According to the graph, what was the combined value, in dollars, of wheat and fruits, nuts, and berries in 1997? **19.6 billion**

Note: Together, the two categories comprise 20% of the whole. 20% of 98 billion is equal to 19.6 billion.
3. The circle graph below represents the raffle ticket sales of five students in the tenth grade. In all, 120 tickets were sold.

Who accounted for about 25% of the sales?

A. Derrick  
B. Zia  
C. Tiffany  
D. Rita  
E. Juan

How many students each sold more than 30 tickets?

A. 1  
B. 2  
C. 3  
D. 4  
E. 5

Note: 30 tickets is ¼ of 120. Only Derrick's section is greater than ¼.
4. The circle graph below represents the lunch items sold in the cafeteria on September 13, 2005.

![Circle graph]

If 200 students bought lunch at the cafeteria, and each student bought one item only, how many students bought hot dogs or pizza?

\[
\left( \frac{20}{100} \cdot 200 \right) + \left( \frac{10}{100} \cdot 200 \right) = 40 + 20 = 60
\]

60 students bought a hot dog or pizza.

According to the above graph, which of the statements is true?

A. Twice as many hamburgers were sold than salad and burritos combined.
B. Three times as many hamburgers were sold than salad and burritos combined.

C. The number of hamburgers sold was equal to the combined number of pizza, salads and hot dogs sold.
D. Half as many hot dogs were sold as burritos.

How many hotdogs and hamburgers were sold? To answer this question, we need to know...

A. The total number of items sold in the cafeteria on that day
B. The total value of sales for the day
C. The total value of hotdogs and hamburgers sold on the day
D. The cost of one hot dog and one hamburger
Scatter plots show, at a quick glance, whether there is a relationship between two sets of data. In other words, it shows whether one variable is affected by another variable.

**Example:** The following scatter-plot shows the relationship between the hours spent preparing for the math section of the CAHSEE and the final math score on the CAHSEE for nine students at Hewett High School.

![Scatter-plot](image)

Note: The lowest possible score on the CAHSEE is 250 and the highest possible score is 450. A passing score is 350.

The relationship between the two variables can be . . .

- **Positive:** The line goes up (as one set of data increases, the other increases as well)
- **Negative:** The line goes down (as one set of data increases, the other decreases)
- **Neutral:** Points appear random with no clear direction (there is no effect of one on the other).

In the above scatter plot, there is a positive relationship between the number of hours spent in preparation for the CAHSEE and the actual score on the CAHSEE.

According to the scatter-plot, the student who received a passing score or better spent at least 20 hours preparing for the test.
vii. **Line of Best Fit**

When there is a positive or negative relationship between data on a scatter plot, we can best represent the relationship by drawing a **straight line** through the data. This line is called a **line of best fit**.

A line of best fit does not necessarily have to pass through every point on the scatter plot; in fact, it may only pass through some of the points. (It can even pass through none of the points!) It simply has to follow the **general direction** of the points on the graph.

Let's return to the scatter plot showing the hours spent preparing for the CAHSEE and the actual scores on the CAHSEE among nine students at Hewett High School:

![Scatter plot](image)

We can draw a **line of best fit** to represent the **trend** in the data:

![Line of Best Fit](image)
1. The scatter plot shows two sets of data: dollars spent on food and dollars spent on clothes.

What conclusions can be drawn from the scatter plot?

A. In general, dollars spent on food increase as dollars spent on clothes increase.
B. In general, dollars spent on food decrease as dollars spent on clothes increase.
C. In general, dollars spent on food remain the same as dollars spent on clothes increase.
D. In general, there is no relationship between dollars spent on food and dollars spent on clothes.
2. The scatter plot shows two sets of data: hours spent gardening and hours spent on sleeping.

What conclusions can be drawn?

A. In general, the number of hours spent gardening increases as the number of hours spent sleeping increases.

B. In general, the number of hours spent gardening decreases as the number of hours spent sleeping decreases.

C. In general, there is no relationship between the number of hours spent gardening and the number of hours spent sleeping.

D. In general, the number of hours spent sleeping is unchanged as the number of hours spent gardening increases.
C. Misleading Graphs

Sometimes the way in which data is presented may be misleading; when this happens, people may draw false conclusions. On the CAHSEE, you will be asked to evaluate statistical claims and determine whether a claim is misleading.

i. Skewing the Scale

Example: The following bar graph shows the telephone bill for two months: April and May.

Based on the bar graph, which of the following conclusions is true?
A. The bill tripled from April to May.
B. **The bill doubled from April to May.**
C. The bill increased by one-half from April to May.
D. The bill increased by one-third from April to May.

Hint: Look at the vertical axis; where does it begin? 50

The way in which the data is presented exaggerates the increase. Always look at the scale of the graph before drawing any conclusions.

Note to Tutors: Point out that, even though the bar representing the cost in May is 3 times bigger than that of April, the actual cost is only twice as high. The bill doubled, rather than tripled. Ask students to re-draw the bar graph to accurately represent the data.
ii. **Exaggerated Claims**

Sometimes, people present survey results in a way that allows them to make exaggerated claims.

**Example:** An independent research company sent a survey to 300,000 people in the Midwest, asking them to choose *one of four* categories of music that they most enjoyed listening to on the radio. The categories consisted of classical, rock, reggae and jazz. The results are shown on the bar graph below.

Survey Results: Genre of Music Listened to on Radio

![Bar Graph]

Based on this graph, the following claim was made: "It is clear that reggae is currently the most popular music in this country"

Can this claim be made with certainty? **No**

Are there other factors that should be considered when making this claim? Explain.

- The survey was conducted in the Midwest only; therefore we cannot draw conclusions for the nation as a whole.
- The survey asked people to rank music they listen to on the radio; it is possible that people prefer to hear one type of music on the radio and other types of music in their homes and at concerts.
- Finally, and most importantly, the survey gave subjects only four categories from which to choose; it is very possible that their favorite type of music did not fall into any of the four categories.
1. Melanie asked each student in her class to rank four colors (green, blue, yellow, red) from favorite to least favorite. She then displayed the results of the survey with a pie graph:

![Pie chart showing color preferences]

From these results, Melanie made the following claim: "Yellow and green are the two favorite colors of the class." Which is the best explanation for why this claim might be misleading?

A. The survey was not representative of every student in her class.

B. **The survey limited the choices to four colors only.**

C. The survey should have been given to students in other classes.

D. The survey should have been given separately to boys and girls.
2. Marsha surveyed all the members of the middle school band about their favorite class this semester. The results are shown in the table below:

**Favorite Class**

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band</td>
<td>18</td>
</tr>
<tr>
<td>English</td>
<td>9</td>
</tr>
<tr>
<td>Math</td>
<td>12</td>
</tr>
<tr>
<td>Science</td>
<td>15</td>
</tr>
</tbody>
</table>

From these results, Marsha concluded that band was the favorite class among all of the students at her school. Which is the best explanation for why her conclusion might not be valid?

A. The survey should have been done each day for a week.

**B. The sample was not representative of all the students at the school.**

C. The survey should have been done with eighth-grade students only.

D. The band meets only 3 days a week.

3. Blake Envelopes produces both personal and business envelopes. The chart below presents the production costs and production output (i.e. the number of envelopes produced) for both products.

<table>
<thead>
<tr>
<th>Product</th>
<th>Total Production Costs for Envelopes</th>
<th>Number of Envelopes Produced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal Envelopes</td>
<td>$4,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Business Envelopes</td>
<td>$6,000</td>
<td>300,000</td>
</tr>
</tbody>
</table>

The CEO examined the chart and concluded that the company is spending more money on business envelopes than on personal envelope. Is he right? Explain.

No; if we look at cost per envelope, Blake Envelopes is spending the same amount on both personal and business envelopes: 2 cents per envelope.
Unit Quiz: The following questions appeared on the CAHSEE:

1. The graph below represents the closing price of a share of a certain stock for each day of the week. Which day had the greatest increase in the value of this stock over that of the previous day?

Answer: Wednesday
Standard: Grade 7, 1.1

2. The circle graph shown below represents the distribution of the grades of 40 students in a certain geometry class. How many students received A’s or B’s?

A. 6
B. 10
C. 15
D. 20

Standard: Grade 7, 1.1
3. Based on the bar graph below, which of the following conclusions is true?

A. Everyone ran faster than 6 meters per second.
B. The best possible rate for the 100-meter dash is 5 meters per second.
C. The first place runner was four times as fast as the fourth place runner.
D. The second place and third place runners were closest in time to one another.

Speed of Four Runners in a 100-Meter Dash

<table>
<thead>
<tr>
<th>Speed (meters per second)</th>
<th>1st place</th>
<th>2nd place</th>
<th>3rd place</th>
<th>4th place</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Runners

Standard: Grade 7, 1.1
4. Three-fourths of the 36 members of a club attended a meeting. Ten of those attending the meeting were female. Which one of the following questions can be answered with the information given?

A. How many males are in the club?
B. How many females are in the club?
C. How many male members of the club attended the meeting?
D. How many female members of the club did not attend the meeting?

Standard: Grade 6, 2.5

5. The Smithburg town library wanted to see what types of books were borrowed most often.

According to the circle graph shown above –

A. more Children’s books were borrowed than Romance and Science fiction combined.
B. more than half of the books borrowed were Children’s, Mysteries, and Art combined.
C. more Mysteries were borrowed than Art and Science Fiction combined.
D. more than half of the books borrowed were Romance, Mysteries, and Science Fiction combined.

Standard: Grade 6, 2.5
6. The cost of a ticket to Funland varies according to the season. Which of the following conclusions about the number of tickets purchased and the cost per ticket is best supported by the scatter plot below?

A. In general, the cost per ticket increases as the number of tickets purchased increases.

B. In general, the cost per ticket is unchanged as the number of tickets purchased increases.

C. In general, the cost per ticket decreases as the number of tickets purchased increases.

D. In general, there is no relationship between the cost per ticket and the number of tickets purchased.

[Scatter plot showing a downward trend in ticket prices as the number of tickets purchased increases.]

Standard: Grade 7, 1.2
7. John drew a graph of his expenses for March, April, and May. During that time, his electric bill stayed about the same, and his gas bill decreased each month. In May, he had to buy new clothes, increasing his expenses for clothing.

Looking at the graph above, what is most likely the meaning of I, II, and III?
A. I = gas, II = clothing, III = electric
B. I = gas, II = electric, III = clothing
C. I = electric, II = clothing, III = gas
D. I = clothing, II = electric, III = gas

8. The table below shows the number of visitors to a natural history museum during a 4-day period.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of Visitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>597</td>
</tr>
<tr>
<td>Saturday</td>
<td>1115</td>
</tr>
<tr>
<td>Sunday</td>
<td>1346</td>
</tr>
<tr>
<td>Monday</td>
<td>365</td>
</tr>
</tbody>
</table>

Which expression would give the BEST estimate of the total number of visitors during this period?
A. 500 + 1100 + 1300 + 300
B. 600 + 1100 + 1300 + 300
C. 600 + 1100 + 1300 + 400
D. 600 + 1100 + 1400 + 400

Standard: Grade 7, 1.1 & MR 1.1
9. Which scatter plot shows a negative correlation?

**Answer:** B  

**Standard:** Grade 7, 1.2
10. What do the data below suggest about the relationship between quality and price?

A. In general, as the price increases, the quality also increases.

B. In general, as the price increases, the quality decreases.

C. In general, as the price increases, the quality remains the same.

D. In general, there is no correlation between quality and price.

Standard: Grade 7, 1.2
11. Using the line of best fit shown on the scatter plot below, which of the following best approximates the rental cost per video to rent 300 videos?

A. $3.00
B. $2.50
C. $2.00
D. $1.50

Note: Extending the line, 300 videos would correspond to about $450, or $1.50 per video.

Standard: Grade 7, 1.2 & MR 2.3
12. The graph below shows the value of Whistler Company stock at the end of every other year from 1994 to 2000.

From this graph, which of the following was the most probable value of Whistler Company stock at the end of 1992?

A. -$10  
B. $1  
C. $10  
D. $20

Standard: Grade 7, 1.1 & MR 2.3

13. Quality Drinks used the data in the table below to create an advertisement. It claims, “Quality Drinks cares. We have one-tenth the number of injuries at work as Best Drinks has.” Why is this claim misleading?

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Injuries at Work</th>
<th>Years in Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Drinks</td>
<td>1000</td>
<td>50</td>
</tr>
<tr>
<td>Quality Drinks</td>
<td>100</td>
<td>2</td>
</tr>
</tbody>
</table>

A. On the average, Best Drinks has more injuries per year  
B. The claim should say, “one-fifth the number of injuries.”  
C. The claim should say, “twenty-five percent fewer injuries.”  
D. **On the average, Quality Drinks has more injuries per year.**

Standard: Grade 6, 2.5
14. Consider the circle graph shown below. How many hours a day does Ramon spend in school?

A. 2 hours

B. 4 hours

C. 6 hours

D. 8 hours

Standard: Grade 7, 1.1
15. After three hours of travel, Car A is about how many kilometers ahead of Car B?

A. 2  
B. 10  
**C. 20**  
D. 25  

Standard: Grade 7, 1.1

16. The graph below shows the time of travel by pupils from home to school. How many pupils must travel for more than 10 minutes?

A. 2  
B. 5  
**C. 7**  
D. 8  

Standard: Grade 7, 1.1
17. The students at a high school were asked to name their favorite type of art. The table below shows the results of the survey.

<table>
<thead>
<tr>
<th>Type of Art</th>
<th>Name of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Painting</td>
<td>714</td>
</tr>
<tr>
<td>Drawing</td>
<td>709</td>
</tr>
<tr>
<td>Sculpture</td>
<td>296</td>
</tr>
<tr>
<td>Other</td>
<td>305</td>
</tr>
</tbody>
</table>

Which circle graph BEST shows these data?

Answer: B

Standard: Grade 7, 1.1
18. The number of games won over four years for three teams is shown on the graph below.

Which statement is true based on this information?

A. Team 3 always came in second.
B. Team 1 had the best average overall.
C. Team 1 always won more games than Team 3.
D. Team 2 won more games each year than in the previous year.

Standard: Grade 6, 2.5
Unit 3: Probability

Probability is a measure of how likely it is that an event will occur.

A. Probability as a Fraction

Probability can be expressed as a fraction between 0 and 1, where 0 represents an event that cannot happen and 1 represents an event that is sure to happen.

Example: The probability that a penny will land on its head is $\frac{1}{2}$:

\[
\text{Probability (Head)} = \frac{1}{2} \quad \text{1 event: heads} \quad \frac{1}{2} \quad \text{Out of 2 equally likely outcomes}
\]

Example: The probability that the die will land on 4 is $\frac{1}{6}$:

\[
\text{Probability (4)} = \frac{1}{6} \quad \text{1 face on the die is 4} \quad \frac{1}{6} \quad \text{6 faces on the die (1, 2, 3, 4, 5, 6)}
\]
B. Probability as Percent

Probability can also be represented as a **percent**: an event that cannot occur has a **0% probability**, while an event that is sure to occur has a **100% probability**.

**Example:** A spinner has four equal sectors: yellow, red, blue, and green. Find the probability that the arrow will stop at yellow.

![Diagram of a spinner with four equal sections: red, yellow, blue, and green.]

Probability (Yellow) = $\frac{1}{4}$ yellow section

$\frac{1}{4}$ sections altogether

Now convert the fraction $\frac{1}{4}$ to a percent:

Percent means **per hundred**, or out of 100. We first convert our fraction $\frac{1}{4}$ to a fraction with a denominator of 100:

$$\frac{1}{4} = \frac{25}{100}$$

Now we can express this as a percent: $\frac{25}{100} = 25\%$

So the probability, expressed as a percent, that the arrow will stop at yellow is **25%**.
### On Your Own: Express each fraction as a percent:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{10}$</td>
<td>10%</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>20%</td>
</tr>
<tr>
<td>$\frac{4}{5}$</td>
<td>80%</td>
</tr>
<tr>
<td>$\frac{3}{10}$</td>
<td>30%</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>75%</td>
</tr>
<tr>
<td>$\frac{7}{20}$</td>
<td>35%</td>
</tr>
<tr>
<td>$\frac{3}{5}$</td>
<td>60%</td>
</tr>
</tbody>
</table>

### Express each percent as a fraction in reduced terms:

<table>
<thead>
<tr>
<th>Percent</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>$\frac{3}{10}$</td>
</tr>
<tr>
<td>60%</td>
<td>$\frac{3}{5}$</td>
</tr>
<tr>
<td>75%</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>25%</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>50%</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>45%</td>
<td>$\frac{9}{20}$</td>
</tr>
<tr>
<td>35%</td>
<td>$\frac{7}{20}$</td>
</tr>
</tbody>
</table>
C. Finding the Probability of an Event Occurring 6, 3.3

i. Equal Likelihood

Given an equal likelihood that each event will occur, the probability of any one event occurring is found by dividing the number of ways the event can occur by the total number of possible events.

\[
\text{Probability of an Event Occurring} = \frac{\text{# of Ways that Event Can Occur}}{\text{Total # of Possible Events}}
\]

Example: A bag contains six marbles. Each one is a different color: green, pink, black, red, purple, and orange. Rachel reaches into the bag and, without looking, picks one marble. What is the probability that she chooses a purple marble?

Steps:

• Begin with the denominator. The denominator is always the total number of possibilities. In this problem, the total number of possible colors that Rachel could have picked in that draw is 6.

• Now find the numerator: The numerator is the number of ways that it is possible to get the particular answer referred to in the problem. In this problem, it asks for the number of ways to get a purple marble. How many ways are there? (In other words, how many purple marbles were in the bag?) 1.

• Now make your fraction: \( \frac{1}{6} \) ←← ←← Numerator ←← ←← Denominator
1. Marla draws one card from a standard deck of cards. What is the probability that she will draw the king of diamonds?

**Note:** A standard deck of cards contains 52 cards. Only one of these cards is the king of diamonds.

Probability of drawing a king of diamonds is \( \frac{1}{52} \)

2. What is the probability of tossing a penny and landing on a tail?

Express answer as a **fraction**: \( \frac{1}{2} \) and as a **percent**: 50%

3. What is the probability of rolling a die (with faces 1 through 6) and landing on a 6?

Answer: \( \frac{1}{6} \)
ii. Unequal Likelihood

There are times when the likelihood of the occurrence of any one event is not equal. Look at the example below.

**Example:** A box contains 8 balls: 3 of the balls are blue; 4 are red; and 1 is purple. Dina reaches into the box and, without looking, picks one ball. Find the probability of picking a red ball.

Although there are 8 balls, the chance of picking any one color is not equal because the number of balls of each color is not equal.

The probability of picking a red ball is higher than the probability of picking either a blue or purple ball because there are 4 red balls in the box, as compared with only 3 blue balls and 1 purple ball.

The probability of picking a red ball is \( \frac{4}{8} \), which is equal to \( \frac{1}{2} \).

Expressed as a percent, the probability is 50%.

The probability of picking a purple ball is \( \frac{1}{8} \) (or 12.5%).

The probability of picking a blue ball is \( \frac{3}{8} \).

Notice that the denominator is always the same. The denominator is the total number of balls in the bag (i.e. the total number of possibilities); that does not change.

It is the numerator that varies. The value of the numerator depends on the color we are focusing on and how many balls of that color exist.
Let's look at another example:

Example: What is the probability that the spinner will stop at red?

Notice that, while the spinner is divided into six equal sections, some colors appear in more than one section.

How many sections are there altogether? 6 ← Denominator

How many red sections are there? 2 ← Numerator

Find the probability that the spinner will stop on red: \( \frac{2}{6} = \frac{1}{3} \)

(Note: Reduce the fraction to lowest terms).

Now express this probability in terms of percent: 33.33%
On Your Own

1. Eleanor rolls a numbered cube with the numbers 1 to 6 on it. What is the probability that the number she rolls will be a prime number?

There are 3 primes between 1 and 6: 2, 3, & 5

Answer: \( \frac{3}{6} = \frac{1}{2} = 50\% \)

2. Tracy rolls a numbered cube with the numbers 1 to 6 on it. What is the probability that the number she rolls will be an even number?

There are 3 even numbers: 2, 4, & 6

Answer: \( \frac{3}{6} = \frac{1}{2} = 50\% \)

3. Erica has 5 blue pens, 3 black pens, and 6 red pens in her desk drawer. If she randomly picks a pen from her drawer, what is the probability that she will pick a black pen?

\( \frac{3 \text{ black pens}}{14 \text{ pens}} = \frac{3}{14} \)
D. Probability of an Event Not Occurring

On the CAHSEE, you will be asked to find the probability of an event not occurring.

Probability of an event not occurring is equal to . . .

$1 - \text{Probability of the event occurring}$

Example: When rolling a die, what is the probability of it not landing on a 2?

There are 6 possible outcomes when rolling a die: 1, 2, 3, 4, 5 and 6.

The probability of landing on 2 is $\frac{1}{6}$.

The probability of not landing on 2 is equal to $1 - \frac{1}{6}$.

In order to subtract 1/6 from 1, we should convert 1 to an equivalent fraction with 6 as the denominator. The number 1 can be converted to a fraction by making the numerator and denominator equal:

$\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$

This is equal to 1.

Therefore, the probability of not landing on 2 is $\frac{5}{6}$. 
Let's do another example together: 6, 3.3

**Example:** When rolling a numbered cube (with the numbers 1 to 6 on it) what is the probability of not landing on an odd number?

How many possible outcomes are there? 6 (Denominator)

How many ways are there to get an odd number? 3 (Numerator)

What is the probability of landing on an odd number? \( \frac{3}{6} = \frac{1}{2} \)

The probability of not landing on an odd number is . . .

\[ 1 - \text{Probability of landing on an odd number} \]

\[ \frac{2}{2} - \frac{1}{2} = \frac{1}{2} \]

**Answer:** \( \frac{1}{2} \), or 50%

**On Your Own**

1. A spinner has four equal sectors: yellow, red, blue, and green. What is the probability that the arrow will stop on a color other than red? Express your answer as a fraction and a percent.

\[ 1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4} \]

or 75%
2. Brenda tosses a coin in the air. What is the probability that the spinner will not land on heads? Express your answer as a fraction and a percent.

\[
\frac{1}{2}
\]

\[
\frac{2}{2} - \frac{1}{2} = \frac{1}{2} \text{ or } 50\%
\]

3. Paul is thinking of a number that is greater than 10 and less than 20. What is the probability that the number is not a prime number? Express your answer as a fraction only.

Possible Prime #'s: 11, 13, 17, 19

Possible #'s: 11, 12, 13, 14, 15, 16, 17, 18, 19

Probability (not a prime) = \(1 - \frac{4}{9} = \frac{5}{9}\)

4. Erica has 5 blue pens, 3 black pens, and 6 red pens in her desk drawer. If she randomly picks a pen from her drawer, what is the probability that she will pick a pen that is not black? Express your answer as a fraction only.

Black pens: 3

Total pens: 14

\[
1 - \frac{3}{14} = \frac{11}{14}
\]
E. Independent Events

Two events are independent if the outcome of the first event does not affect the outcome of the second event.

Let's begin by comparing two scenarios:

**Example:** Dina tosses a penny and it lands on heads. She tosses the penny a second time and, again, it lands on heads. If she tosses the coin a third time, what is the theoretical probability that it will land on heads?

To solve this problem, we need to know that each time we toss a coin, we have an equal chance of landing on heads or tails.

No matter how many times we toss the coin, and no matter how many times it lands on heads, the probability of it landing on heads again remains 50%.

The fact that we land on heads two times in a row neither increases nor decreases the probability of landing on heads the third time. There is an equal chance of landing on heads each time.

**Note:** The problem asks us to find the theoretical probability of an event. The term "theoretical probability" may appear on the CAHSEE. Do not be confused by this term. It's just a fancy way of saying the probability based on probability principles. Just apply the principles we're learning here!
Now compare the last problem with this next problem:

**Example:** Dina tosses a penny twice. What is the probability that it will land on **heads both times**?

This is a very different problem than the previous one. Here, we are asked to find the probability of **two independent events occurring in sequence**. This is often referred to as **conditional probability**: the probability that a second event will occur, **given** (or **on condition**) that a first event has occurred:

"**Given A...**, what are the chances of **B...**" 

Each time we toss the penny, there are two possible outcomes. We can look at all of the possible outcomes for two separate tosses by using a **tree diagram**. A **tree diagram** uses branches to show every possible outcome for an event:

To find the probability of the penny landing on its head both times, we **multiply** the probability of it landing on its head the first time ($\frac{1}{2}$) **by** the probability of it landing on its head the second time ($\frac{1}{2}$):
Let's look at another example: 6, 3.1 & 6, 3.3

**Example:** Mr. and Mrs. Smith want to start a family. They would like to have three children, and hope that each child is a girl. What is the probability that this will occur?

We can represent the possible outcomes with a tree diagram:

![Possible Genders of 3 Children](image_url)

We can see from the tree diagram that there are **two** possible outcomes for **each** child: Boy or Girl.

To find the probability that they will have three girls in a row, we need to **multiply** the probability of having a girl the **first** time by the probability of having a girl the **second** time by the probability of having a girl the **third** time:

\[
\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}
\]

Notice that in the last row, there are **8** branches. This number is also the denominator of the fraction.
Look at the tree diagram again:

What is the probability that the first child will be a girl, the second child a girl, and the third child a boy?

\[
\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}
\]

What is the probability that all three children will be boys?

\[
\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}
\]
We can also use a table to illustrate problems involving conditional probability:

**Example:** Patty tosses a penny and then rolls a die. What is the probability that the penny will land on tails and the die will land on a 4?

The table below shows all of the possible outcomes for tossing a penny and rolling a die:

```
<table>
<thead>
<tr>
<th>Die</th>
<th>Penny</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H1</td>
</tr>
<tr>
<td>1</td>
<td>T1</td>
</tr>
<tr>
<td>2</td>
<td>H2</td>
</tr>
<tr>
<td>2</td>
<td>T2</td>
</tr>
<tr>
<td>3</td>
<td>H3</td>
</tr>
<tr>
<td>3</td>
<td>T3</td>
</tr>
<tr>
<td>4</td>
<td>H4</td>
</tr>
<tr>
<td>4</td>
<td>T4</td>
</tr>
<tr>
<td>5</td>
<td>H5</td>
</tr>
<tr>
<td>5</td>
<td>T5</td>
</tr>
<tr>
<td>6</td>
<td>H6</td>
</tr>
<tr>
<td>6</td>
<td>T6</td>
</tr>
</tbody>
</table>
```

Note: "H1" means that the penny lands on heads and the die lands on 1.

How many possible outcomes are there? 12

What is the probability of each outcome? \( \frac{1}{12} \)

What is the probability of landing on tails and rolling a 2? \( \frac{1}{12} \)

**Note:** We can also solve the problem through multiplication:

Probability of landing on tails = \( \frac{1}{2} \)

Probability of rolling a 2 = \( \frac{1}{6} \)

Multiply the two probabilities:

\[
\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}
\]

Answer: \( \frac{1}{12} \)
ii. Conditional Probability with a Twist  6, 3.1 & 6, 3.3

Look at the following example:

**Example:** Julien is rolling a numbered cube with the numbers 1 to 6 on it. He rolls the cube twice. What is the probability that the two rolls will have a sum of 8?

To answer this question, we must first find the number of possible ways to get a sum of 8 when two number cubes are rolled. We can solve this with a table.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A check mark is placed in each square that gives us a sum of 8.

We see that there are 5 possible ways to get a sum of 8:

2 & 6  3 & 5  4 & 4  5 & 3  6 & 2

We can also see from the chart that there are 36 squares; therefore, there are 36 possible outcomes when a cube is rolled twice.

Probability that the two rolls will have a sum of 8: \( \frac{5}{36} \)

Now find the probability that a numbered cube, when rolled twice, will have a sum of 6:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer: \( \frac{5}{36} \)
On Your Own 6, 3.1 & 6, 3.3

1. Jennifer rolled a pair of dice. What is the probability that she rolled a 1 on the first die and a 4 on the second die?

\[
\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}
\]

2. Jennifer rolled a die and it landed on a 6. She then rolled a second die. What is the theoretical probability that she will land on 6 again?

\[
\frac{1}{6}
\]

3. Jennifer rolls a die twice. What is the probability that she will land on 6 both times?

\[
\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}
\]
4. Kevin tosses a penny in the air two times. What is the probability that the penny will land on tails the first time and heads the second time?

\[
\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
\]

5. A spinner has four equal sectors: yellow, red, blue, and green. Sheila spins the arrow three times. What is the probability that the arrow will land on red the first time, yellow the second time, and blue the third time?

\[
\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}
\]

6. A spinner has four equal sectors: yellow, red, blue, and green. Jeffrey spins the arrow and it lands on yellow. He spins the arrow a second time and it lands on yellow again. If he spins the arrow a third time, what is the probability that the arrow will land on yellow again?

\[
\frac{1}{4}
\]
7. A spinner has four equal sectors: yellow, red, blue, and green. Jeffrey spins the arrow twice. What is the probability that he will land on yellow both times?

\[
\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}
\]

8. A spinner has four equal sectors: yellow, red, blue, and green. The arrow is spun twice. What is the probability that the arrow will land on the yellow region on the first spin and on the red region on the second spin?

\[
\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}
\]

9. Max enters a three-digit number in the lottery. The number he enters is 343. That night, when he watches the 10:00 news, he learns that the winning number is 459. He tells his brother that he will play the lottery again next week and play the winning number (459). His brother tells him that 459 has a lower chance of coming up a second time. Is he right? Explain.

**No; his brother is wrong.** The theoretical probability that any one number will be drawn is the same each time. There is neither a greater chance nor a lesser chance that the number 459 will be drawn.
F. Dependent Events

Two events are **dependent** if the outcome of the first event **affects** the outcome of the second event, so that the probability is changed.

Let's begin by **comparing two problems**. In the **first** problem the **outcome of the first event does not affect the outcome of the second event**:

**Example:** Ariel drew from a standard deck of cards and picked a king. He *replaced* the king and drew a second card. What is the probability that he will pick a king again?

Since Ariel *replaces* the king in the deck of cards, he has an **equal chance** of picking a king a second time.

There are 52 possible cards that Ariel can draw, and there are 4 possible ways of getting a king:

\[
\text{Possible Ways to Drawing a King} = \frac{4}{52} = \frac{1}{13}
\]
Now let's look at the **second** problem.

**Example:** Ariel drew from a standard deck of cards and picked a king. *Without replacing the king,* he drew a second card. What is the probability that he drew another king?

In this problem, Ariel **does not replace the king.** Both the size of the larger sample (the total number of cards) and the size of the sub-sample (total number of kings) has now changed. There are now only 51 cards and the number of kings has decreased from 4 to 3:

<table>
<thead>
<tr>
<th># of Ways to Draw a King</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total # of Possible Cards</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>

**Note:** The outcome of the first event has changed the outcome of the second event.

**On Your Own:**

1. Erica has 5 blue pens, 3 black pens, and 6 red pens in her desk drawer. She reaches into her drawer and, without looking, picks a black pen. She keeps the black pen and then reaches into her drawer again to pick a second pen. What is the probability that the second pen will also be black?

<table>
<thead>
<tr>
<th># of Possible Ways to Pick a Black Pen</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total # of Possible Outcomes</td>
<td>13</td>
</tr>
</tbody>
</table>

What if Erica had replaced the black pen on the first try? What would be the probability that she would pick the black pen again?

<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
</tr>
</tbody>
</table>
2. A box contains 8 balls: 3 of the balls are blue; 4 are red; and 1 is purple. Dina reaches into the box and, without looking, picks a red ball. She does not put the ball back in the box. Marvin then reaches into the bag and removes a second ball. What is the probability that the ball that Marvin picks is blue?

\[ \frac{3}{7} \]

3. A bag contains six marbles. Each one is a different color: green, pink, black, red, purple, and orange. Rachel reaches into the bag and removes a purple marble. Without replacing it, she reaches into the bag again and removes a second marble. What is the probability that this second marble is orange? Express the answer as a fraction (in lowest terms) and a percent.

\[ \frac{1}{5} \text{ or } 20\% \]
4. Debra has a box containing 7 green gum balls and 3 blue gum balls. She reaches into the box and, without looking, picks a green gum ball. She keeps it and reaches into the box again. What is the probability that she will draw a blue gum ball? Express the answer as a fraction (in lowest terms) and a percent.

\[ \frac{3}{9} = \frac{1}{3} \text{ or 33\%} \]

5. Debra has a box containing 7 green gum balls and 3 blue gum balls. She reaches into the box and without looking, picks a green gum ball. She puts it back and then reaches into the box again. What is the probability that Debra will pick a blue gum ball? Express the answer as a fraction (in lowest terms) and a percent.

\[ \frac{3}{10} \text{ or 30\%} \]

For reinforcement and/or spiraling, play "Game of High/Low" with your students. (See the packet, "Cooperative Learning Activities to Build Reasoning Skills.")
Unit Quiz: The following problems appeared on the CAHSEE.

1. A paper bag contains 2 red balls, 1 green ball, and 2 yellow balls. If Pat takes one ball out of the bag without looking, what is the probability that it is red?

   Answer: $\frac{2}{5}$  
   Standard: Grade 6, 3.1

2. To get home from work, Curtis must get on one of the three highways that leave the city. He then has a choice of four different roads that lead to his house. In the diagram below, each letter represents a highway, and each number represents a road.

   If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?

   A. $\frac{1}{16}$

   B. $\frac{1}{12}$

   C. $\frac{1}{4}$

   D. $\frac{1}{3}$  
   Standard: Grade 6, 3.1
3. What is the probability that the arrow will not stop on red if you spin it one time? (Assume that the spinner is fair.)

A \( \frac{1}{4} \)

B \( \frac{1}{3} \)

C \( \frac{3}{4} \)

D \( \frac{4}{3} \)

4. A bucket contains 3 bottles of apple juice, 2 bottles of orange juice, 6 bottles of tomato juice, and 8 bottles of water. If Kira randomly selects a bottle, what is the probability that she will select a drink other than water?

A \( \frac{3}{4} \)

B \( \frac{11}{19} \)

C \( \frac{8}{19} \)

D \( \frac{1}{4} \)
5. Heather flipped a coin five times, and each time it came up heads. If Heather flips the coin one more time, what is the theoretical probability that it will come up tails?

A \( \frac{1}{6} \)  
B \( \frac{1}{2} \)  
C \( \frac{3}{5} \)  
D \( \frac{5}{6} \)  

6. Mr. Gulati is holding five cards numbered 1 through 5. He has asked five students to each randomly pick a card to see who goes first in a game. Whoever picks the card numbered 5 goes first. Juanita picks first, gets the card numbered 4, and keeps the card. What is the probability that Yoko will get the card numbered 5 if she picks second?

A \( \frac{1}{2} \)  
B \( \frac{1}{3} \)  
C \( \frac{1}{4} \)  
D \( \frac{1}{5} \)
7. A bag contained four green balls, three red balls, and two purple balls. Jason removed one purple ball from the bag and did not put the ball back in the bag. He then randomly removed another ball from the bag. What is the probability that the second ball Jason removed was purple?

A \[ \frac{1}{36} \]  
B \[ \frac{1}{9} \]  
C \[ \frac{1}{8} \]  
D \[ \frac{2}{9} \]  

8. Rosella is rolling a numbered cube with the numbers 1 to 6 on it. She rolls the cube twice. What is the probability that the two rolls will have a sum of 10?

Answer: \[ \frac{1}{12} \]  

Standard: Grade 6, 3.3
9. Carmen wants to buy a new car. Her choices are a 2-door or a 4-door, a convertible top or a hard top, and red, white, or black. Which of the following tree diagrams represents all of the choices for the car?

---

**Answer: D**  
**Grade 6, Standard 3.1**