CAHSEE on Target
UC Davis, School and University Partnerships

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CAHSEE on Target
UC Davis School University Partnerships
Answer Key: Geometry/Measurement

Introduction to the CAHSEE

The CAHSEE stands for the California High School Exit Exam. The mathematics section of the CAHSEE consists of 80 multiple-choice questions that cover 53 standards across 6 strands. These strands include the following:

Number Sense (14 Questions)
Statistics, Data Analysis & Probability (12 Questions)
Algebra & Functions (17 Questions)
Measurement & Geometry (17 Questions)
Mathematical Reasoning (8 Questions)
Algebra 1 (12 Questions)

What is CAHSEE on Target?

CAHSEE on Target is a tutoring course specifically designed for the California High School Exit Exam (CAHSEE). The goal of the program is to pinpoint each student’s areas of weakness and to then address those weaknesses through classroom and small group instruction, concentrated review, computer tutorials and challenging games.

Each student will receive a separate workbook for each strand and will use these workbooks during their tutoring sessions. These workbooks will present and explain each concept covered on the CAHSEE, and introduce new or alternative approaches to solving math problems.

Geometry & Measurement: What’s on the CAHSEE?

On the CAHSEE, you will compare lengths, weights, time intervals and capacities within and across different systems (such as miles per hour or feet per second) and use ratios to solve problems. You will also determine the measure of one-, two- and three-dimensional closed figures, called polygons. These topics are presented as separate units in this workbook.
What is Geometry & Measurement?

The word *geometry* comes from two Greek words:
- *geo* - which means Earth
- *metron* - which means to measure

So geometry means the "measure of the earth." It is a branch of mathematics that deals with the study of points, lines, angles, and shapes (as opposed to the study of numbers).

**Measurement** is the process of quantifying the properties of an object (How big? How small? How long? How wide? How heavy?), using a **standard unit** (inches, feet, meters, pounds, ounces).

In order to measure an object, we must do three things:

1. **Pick** the attribute you want to measure (such as the length, the width, the perimeter, the area, the volume, the weight, the surface area).

2. **Choose** an appropriate **unit of measurement** (such as inches, feet, meters, pounds, ounces)

3. **Measure it**: Find the **number of units** for that object.

**Example:** Maurice would like to know how long his desk is. This involves the following steps:
1. Pick the attribute of the desk to measure: **length**
2. Choose the unit of measure: combination of **feet & inches**
3. Measure the length of the desk: Use a ruler to find the **number of feet** and **inches**.

**On Your Own:**

Evelyn would like to know how heavy her puppy is. She must....

1. Determine which attribute to measure: **weight**
2. Choose an appropriate unit of measure: **pounds & ounces**
3. Measure it.
Unit 1: Converting Units of Measure 1.1

In life, we use conversions all the time.

**Example:** Eric is 6 feet tall. Find his height in inches.

To answer this question, you need to **convert** from feet to inches. Before you can do this, however, you need to know how many inches there are in 1 foot. In other words, you need to know the **conversion factor**.

A **conversion factor** is a numerical factor by which a quantity expressed in one unit must be **multiplied** in order to convert it to another unit. It is often stated as an **equation**:

**Example:** 12 inches = 1 foot

A conversion factor can also be expressed as a **fraction**:

\[
\frac{12 \text{ inches}}{1 \text{ foot}} \quad \text{Conversion factor expressed as a fraction.}
\]

Notice that this fraction is equal to 1 because 12 inches = 1 foot.

So, to find Eric's height in inches, we **multiply** the **quantity we are given** (6 feet) by the **conversion factor**:

\[
6 \text{ feet} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}
\]

We can cancel out common units!

\[
6 \cdot 12 \text{ inches} = 72 \text{ inches}
\]

Eric is 72 inches tall.

On the CAHSEE, you will be asked to convert from **one unit of measure** to another. To do this, you will need to know some common **conversions factors**.


## Conversion Factors

<table>
<thead>
<tr>
<th>Length</th>
<th>Weight</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 inches = 1 foot*</td>
<td>16 ounces = 1 pound*</td>
<td>8 fluid ounces = 1 cup</td>
</tr>
<tr>
<td>3 feet = 1 yard*</td>
<td>2000 pounds = 1 ton*</td>
<td>4 quarts = 1 gallon*</td>
</tr>
<tr>
<td>5,280 feet = 1 mile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 meter = 1,000 millimeters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Means that you will need to **memorize** these conversions for the CAHSEE. The others will probably be given to you.

You should also know the **standard abbreviations** and **symbols for units of measure**:

<table>
<thead>
<tr>
<th>Length</th>
<th>Weight</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foot: ft (')</td>
<td>Pound: lb</td>
<td>Cup: c</td>
</tr>
<tr>
<td>Yard: yd</td>
<td>Gram: g</td>
<td>Quart: qt</td>
</tr>
<tr>
<td>Mile: mi</td>
<td>Kilogram: kg</td>
<td>Gallon: gal</td>
</tr>
<tr>
<td>Meter: m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centimeter: cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Millimeter: mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kilometer: km</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You should know these numerical **prefixes** for the CAHSEE:

- **Deka-** (or **Dec-**) → ten
- **Deci-** → tenth
- **Cent-** → hundred
- **Centi-** → hundredth
- **Kilo-** (or **mill-**) → thousand
- **Milli-** → thousandth

**Note:** You should also know standard conversion factors for **time**:

- 1 hour = 60 minutes
- 1 minute = 60 seconds
Let's look at a problem together:

**Example:** How many inches in 20 feet?

**Steps:**
- **Write** down the **quantity given** in the problem: **20 feet**
- **Choose** the appropriate **conversion factor**: **12 inches = 1 foot**
- **Write** the conversion factor as a **fraction**:

  \[
  \frac{12 \text{ inches}}{1 \text{ foot}} \quad \text{Note: The fraction is equal to 1 (12 in = 1 foot)}
  \]

- **Multiply** the **original quantity** by the **conversion factor**.

  We can place the original quantity over 1 so that we have two fractions. Any whole number can be converted to a fraction by placing it over 1!

  \[
  \frac{20 \text{ ft} \cdot 12 \text{ in}}{1 \cdot 1 \text{ ft}}
  \]

  Since this fraction is equal to 1, we are really just multiplying 20 ft by 1!

  \[
  \frac{20 \text{ ft} \cdot 12 \text{ in}}{1 \cdot 1 \text{ ft}} \quad \text{Notice that we can cancel out common units.}
  \]

  \[
  \frac{20 \cdot 12 \text{ in}}{1 \cdot 1} = 240 \text{ inches}
  \]

**Answer:** There are **240** inches in 20 feet
How to Set Up Your Fraction

In the above, problem, we could also have written the fraction (in the third step) as follows:

\[
\frac{1 \text{ foot}}{12 \text{ inches}}
\]

This fraction is also equal to 1 (1 foot = 12 inches).

However, we always want the denominator of the fraction to have the same unit as that of the quantity given in the problem so that we can cancel out the common units, and therefore simplify the problem. Since the problem asked us to convert 23 feet, we need feet in the denominator so that we are left with inches in the answer.

Example: How many yards are there in 15 feet?

Steps:

• **Write** down the quantity given in the problem: 15 feet.

• **Choose** the appropriate conversion factor: 3 feet = 1 yard.

• **Write** the conversion factor as a fraction:

\[
\frac{1 \text{ yard}}{3 \text{ feet}} \quad \text{Feet in Denominator}
\]

• **Multiply** the original quantity by the conversion factor:

\[
\frac{\cancel{5} 15 \text{ feet} \cdot \frac{1 \text{ yard}}{1 \cancel{3} \text{ feet}}} = \frac{15}{1} \cdot \frac{1}{1} \text{ yds}
\]

\[
= \frac{15}{1} \cdot \frac{1}{1} \text{ yds}
\]

\[
= \frac{15}{1} \cdot \frac{1}{1} \text{ yds}
\]

\[
= \frac{15}{1} \cdot \frac{1}{1} \text{ yds}
\]

Answer: There are 5 yards in 15 feet.
Look at the next example:

Example: How many pounds are there in 3¼ tons?

Steps:

• **Write** down the **quantity given** in the problem: 3¼ tons

• **Choose** the appropriate **conversion factor**: 2000 lbs. = 1 ton

• **Write** this conversion factor as a **fraction**:

  **Note:** We can write this as a fraction in 2 ways:

  \[
  \frac{2000 \text{ lbs}}{1 \text{ ton}} \quad \text{or} \quad \frac{1 \text{ ton}}{2000 \text{ lbs}}
  \]

  Both are equal to 1.

Since we are given the unit of tons in the problem, we want tons to appear in the denominator of the fraction so we can cancel out common units:

Which fraction should we use? \( \frac{2000 \text{ lbs}}{1 \text{ ton}} \)

• **Multiply** the **original quantity** by the **conversion factor**:

  \[
  3\frac{1}{4} \text{ ton} \cdot \frac{2000 \text{ lbs}}{1 \text{ ton}} \rightarrow \text{Write the numerator (with unit)}
  \]

  Solve: \( 3\frac{1}{4} \cdot 2000 \text{ lbs} \rightarrow \frac{13}{4} \cdot 2000 = 6,500 \text{ lbs} \)

  **Answer:** There are 6,500 pounds in 3¼ tons.
On Your Own: 1.1

1. James bought 20 pounds of coffee. How many ounces of coffee did he buy?

Write down the quantity given in the problem: 20 lbs.

Choose the appropriate conversion factor: 16 oz. = 1 lb.

Write conversion factor as fraction: \[
\frac{16 \text{ oz}}{1 \text{ lb}}
\]

Multiply the original quantity by the conversion factor:

\[
20 \text{ lbs} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} = 320 \text{ oz.}
\]

2. Tiffany bought 20 ounces of coffee. How many pounds did she buy?

Write down the quantity given in the problem: 20 oz.

Choose the appropriate conversion factor: 16 oz. = 1 lb.

Write conversion factor as fraction: \[
\frac{1 \text{ lb}}{16 \text{ oz}}
\]

Multiply the original quantity by the conversion factor:

\[
\frac{20 \text{ oz}}{1 \text{ lb}} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{20}{16} \text{ lb} = 1 \frac{1}{4} \text{ lb}
\]
3. Mr. Adam buys 18 quarts of milk each week for his 5 children. How many gallons of milk does this equal?

4½ gallons

4. Rita needs 6 cups of milk for a pie recipe. How many fluid ounces of milk does this represent?

48 fluid ounces

5. Philip ran 25 yards in 40 seconds. How many feet did he run?

75 ft

6. Sam walked half of a mile. How many feet did he walk?

2640 ft
7. How many yards in 21 feet?

7 yards

8. How many inches in 4 feet?

48 inches

9. Greta spent 9 hours studying for the geometry portion of the CAHSEE. How many minutes did she study?

540 minutes

10. David can get to Rachel's house in exactly 11 minutes. Express this time in seconds.

660 seconds
Comparing Measures

On the CAHSEE, you may be asked to compare two or more quantities that are expressed in different units:

Example: Which is bigger: 16 feet or 4 yards?

Steps:

- To answer this question, we need a common unit of comparison. In general, it is easiest to convert all measures to the smallest unit. In this case, that unit is feet. Write down the conversion factor:

  \[3 \text{ feet} = 1 \text{ yard}\]

- We can write this conversion factor as a fraction. Since the number we are converting is in yards, we want to write the fraction so that yards is in the denominator (so we can cancel out units):

  \[
  \frac{3 \text{ feet}}{1 \text{ yard}} \quad \text{Yards in Denominator}
  \]

- Now multiply the number we are converting by the conversion factor:

  \[
  4 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = 12 \text{ feet}
  \]

  **Answer:** There are 12 feet in 4 yards.

- Now, we can compare the two measures:

  \[16 \text{ feet} > 4 \text{ yards}\]
Example: Order the following lengths, from smallest to largest:

- 5'  
- 35"  
- 3' 20"

Notice that the measures are expressed in different units. The first is expressed in feet ('), the second in inches ("), and the third in both feet (') and inches ("").

- To solve this problem, we must use a common unit of comparison. In general, it is easiest to convert all measures to the smallest unit. In this case, that unit is inches. Write down the conversion factor:

  \[12" = 1'\]

- Now, to convert 5' to inches, we multiply it by the conversion factor written as a fraction (with feet in the denominator):

  \[
  \begin{align*}
  5 \text{ ft} \cdot & \frac{12 \text{ in}}{1 \text{ ft}} \\
  & = 60 \text{ in}
  \end{align*}
  \]

- To convert 3' 20" to inches, let's first convert the 3 feet and then add the 20 inches at the end:

  \[
  \begin{align*}
  3 \text{ ft} \cdot & \frac{12 \text{ in}}{1 \text{ ft}} \\
  & = 36 \text{ in} \\
  3' 20" & = 36 \text{ in} + 20 \text{ in} = 56 \text{ in}
  \end{align*}
  \]

- Now, we can compare all three measures:

  \[
  \begin{array}{ccc}
  \text{Smallest} & \text{3'20" = 56 in} & \text{Largest} \\
  35" & 5' = 60 \text{ in}
  \end{array}
  \]
On Your Own:

1. Order the following measures from smallest to largest:

   15,050 feet  \hspace{1cm} 1\frac{1}{2} \text{ miles}  \hspace{1cm} 1 \text{ mile, 1500 feet}

   \textbf{Note: 5,280 feet (') = 1 mile}

   A. Convert to common unit of measurement: feet

   \begin{align*}
   15,050 \text{ ft} & \hspace{1cm} 1\frac{1}{2} \text{ mi} = 7,920 \text{ ft} & 1 \text{ mi, 1500 ft} = 6,780 \text{ ft}
   \end{align*}

   B. Order from smallest to largest:

   \begin{align*}
   1\text{mi, 1500 ft} & \hspace{1cm} 1\frac{1}{2} \text{ mi} & 15,050 \text{ ft}
   \end{align*}

   Smallest \hspace{2cm} \text{Largest}
2. Order the following weights from smallest to largest:

- 5 lb
- 45 oz
- 4 lb 6 oz

A. Convert to common unit of measurement: ounces

- $5 \text{ lb} = 80 \text{ oz}$
- $45 \text{ oz}$
- $4 \text{ lb 6 oz} = 70 \text{ oz}$

B. Order from smallest to largest:

- 45 oz
- 4 lb 6 oz
- 5 lb

Smallest

Largest
3. The table below shows the departure and arrival times for four different trains from Sacramento to Santa Barbara.

<table>
<thead>
<tr>
<th>Departure Time</th>
<th>Arrival Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:30 AM</td>
<td>2:15 PM</td>
</tr>
<tr>
<td>11:45 AM</td>
<td>3:30 PM</td>
</tr>
<tr>
<td>12:30 PM</td>
<td>3:40 PM</td>
</tr>
<tr>
<td>10:45 PM</td>
<td>2:50 AM</td>
</tr>
</tbody>
</table>

Which train takes the **shortest** amount of time?

**Hint:** Figure out the travel time for each train. Then compare.

**Answer:** The flight departing at 12:30 p.m. and arriving at 3:40 p.m. (It will take 3 hours and 10 minutes.)

4. Wanda is 56 inches tall. Express her height in feet and inches.

**Answer:** 4' 8"

5. Miriam spent 450 minutes studying for her final exam in algebra. Express the time in hours and minutes.

**Answer:** 7 ½ hours
Unit Quiz: The following problems appeared on the CAHSEE.

1. Maria rode her bike $1 \frac{1}{4}$ miles. How many feet did she ride on her bike? (5,280 feet = 1 mile)
   A. 6,340
   B. 6,600
   C. 7,180
   D. 7,392

2. One cubic inch is approximately equal to 16.38 cubic centimeters. Approximately how many cubic centimeters are there in 3 cubic inches?
   A. 5.46
   B. 13.38
   C. 19.38
   D. 49.14

3. One millimeter is ____.
   A. $\frac{1}{1000}$ of a meter
   B. $\frac{1}{100}$ of a meter
   C. 100 meters
   D. 1000 meters
4. The table below shows the flight times from San Francisco (S.F.) to New York (N.Y.).

<table>
<thead>
<tr>
<th>Leave S.F. Time</th>
<th>Arrive N.Y. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30 AM</td>
<td>4:50 PM</td>
</tr>
<tr>
<td>12:00 noon</td>
<td>8:25 PM</td>
</tr>
<tr>
<td>3:30 PM</td>
<td>11:40 PM</td>
</tr>
<tr>
<td>9:45 PM</td>
<td>5:50 AM</td>
</tr>
</tbody>
</table>

Which flight takes the longest?
A. The flight leaving at 8:30 a.m.
B. The flight leaving at 12:00 noon
C. The flight leaving at 3:30 p.m.
D. The flight leaving at 9:45 p.m.

5. A boy is two meters tall. About how tall is the boy in feet (ft) and inches (in)? (1 meter = 39 inches)
A. 5 ft 0 in.
B. 5 ft 6in.
C. 6 ft 0 in.
D. 6 ft 6 in.

6. Juanita exercised for one hour. How many seconds did Juanita exercise?
A. 60
B. 120
C. 360
D. 3,600
Unit 2: Scale Drawings

Scale drawings are **reduced or enlarged** representations of **actual** figures or objects. The scale of the drawing is the **ratio** of the size of the drawing to the actual size of the object, and each side of the scale drawing is **proportional** to that of the actual figure.

On the CAHSEE, you will be presented with a scale drawing that represents a **two-dimensional** figure (length and width). You will be given the **ratio** of the size of the drawing to the actual size of the figure and asked to apply this ratio to find the dimensions (or one of the dimensions) of the actual figure.

**Example:** In a scale drawing of a classroom, the length is 2" and the width is 1½ inches. If every scale inch represents 10 yards, what are the actual dimensions of the classroom?

There are two methods that we can use to solve this problem:

- **With Algebra**
- **Without Algebra**
Method I: With Algebra

We can solve a scale problem algebraically, by first setting up a proportion.

A proportion is two equivalent ratios, written as fractions.

Let's look at the problem again:

Example: In a scale drawing of a classroom, the length is 2" and the width is 1½ inches. If every scale inch represents 10 yards, what are the actual dimensions of the classroom?

Let's solve for one dimension at a time.

A. Length

We can represent the relationship between the length of the actual classroom and that of the scale drawing by a proportion, consisting of two equal ratios. Since we are asked to find the actual length of the classroom, this will be our unknown number. We can use a letter, such as $x$, to represent the unknown number.

Note: The units of the numerators must be the same, and the units of the denominators must be the same. In the example above, we will set the unit of the numerator as inches and the unit of the denominator as yards:

\[
\frac{\text{inches}}{\text{yards}} = \frac{\text{inches}}{\text{yards}}
\]

1 inch $= 2$ inches
10 yards $\times$ yards

\[1x = 20 \quad \text{Cross multiply}\]
\[x = 20 \quad \text{The actual length of the room is 20 yards.}\]
B. Width

Let's look at this problem again and solve for the actual width of the classroom:

**Width:**

Let \( y \) = the actual width of the classroom

\[
\frac{1}{10 \text{ yards}} = \frac{1.5 \text{ inches}}{y \text{ yards}}
\]

\[
y = \frac{15}{1} \quad \text{Cross multiply}
\]

\[
y = 15 \quad \text{The actual width of the room is 15 yards.}
\]

**Method II: Without Algebra**

We can also solve this problem without algebra. Just apply the ratio of the actual size to the scale size.

**Length**

Scale: 1 scale inch represents 10 yards

Since every inch on the drawing represents 10 actual yards, then 2 scale inches = \( 2 \cdot 10 \) actual yards.

\[
2 \cdot 10 = 20 \quad \text{The actual length of the classroom is 20 yards.}
\]

**Width**

Scale: 1 scale inch represents 10 yards

Since every inch on the drawing represents 10 actual yards, then 1½ scale inches = \( 1.5 \cdot 10 \) actual yards.

\[
1.5 \cdot 10 = 15 \quad \text{The actual width of the room is 15 yards.}
\]
Let's look at another example:

**Example:** The figure below represents a wall hanging. If the scale is 1 inch = 18 inches, find the length of the actual hanging.

![Wall hanging diagram](image)

**Method 1: Using Algebra**

**Set Up a Proportion & Solve**

\[
\frac{1}{18} = \frac{6}{x} \quad \text{Set up your proportion: } \frac{\text{scale}}{\text{actual}} = \frac{\text{scale}}{\text{actual}}
\]

\[x = 108 \quad \text{Cross multiply}
\]

**Method II: Without Algebra**

Every inch on the scale drawing represents **18 actual inches**.

So \(6 \text{ scale inches} = 6 \cdot 18 \text{ actual inches}\).

The length of the actual figure is **108 inches**.
On Your Own: 1.2

The drawing below represents an actual figure. If the scale is 1 inch = 18 inches, find the width of the actual figure.

With Algebra:

\[
\frac{1}{18} = \frac{5}{x}
\]

\[x = 90\]

Without Algebra:

Since every inch on the drawing represents 18 actual inches, then 5 scale inches = 5 X 18 (or 90) actual inches.
Practice: Use either method to solve the problems below.

1. If on a scale drawing 48 actual feet are represented by 12 scale inches, then $\frac{1}{4}$ scale inches represent how many feet?

   1 foot

2. Marissa has drawn a scale drawing of a tractor trailer. 1 inch in the scale drawing stands for 3 actual feet. How long is the tractor's bumper if it is 2½ inches in the scale drawing?

   1" = 3' so 2½" = 7.5'

3. 1 cm on a map represents a distance of 250 km. The distance between points A & B is 2.7 cm on the map. What is the actual distance between these two points?

   675 km
Unit Quiz: The following problem appeared on the CAHSEE.

1. The actual width \((w)\) of a rectangle is 18 centimeters (cm). Use the scale drawing of the rectangle to find the actual length \((l)\).

![Diagram of rectangle with scale drawing](image)

A. 6 cm  
B. 24 cm  
C. 36 cm  
D. 54 cm  


2. The scale drawing of the basketball court shown below is drawn using a scale of 1 inch (in) = 24 feet (ft).

![Diagram of basketball court](image)

What is the length, in feet, of the basketball court?  
A. 90 ft  
B. 104 ft  
C. 114 ft  
D. 120 ft
Unit 3: Rates

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance or completing a certain task.

Example: A car that travels 60 miles in one hour travels at the rate of 60 miles per hour.

Ratios compare two quantities that are expressed in different units of measure, such as miles per hour. Ratios can be represented as fractions:

Example: 60 miles per hour can be represented as a fraction:

\[
\frac{60 \text{ mi}}{1 \text{ hr}} \quad \text{distance (in miles)} \quad \text{time (in hours)}
\]

In general, when creating a ratio to represent the rate, the distance is the numerator and the time is the denominator.

Expressing Rate in Lowest Terms

Whenever we represent a relationship in terms of rate, we express it in lowest terms.

Example: A man runs 5 miles in 60 minutes. Find his running rate.

Note: This means, "How long does it take him to run 1 mile?"

First, let's write this as a ratio: \( \frac{5 \text{ mi}}{60 \text{ min}} \)

Is this fraction in lowest terms? No

If not, reduce it to its lowest terms: \( \frac{1 \text{ mi}}{12 \text{ min}} \)

Answer: The man can run 1 mile in 12 minutes.
On Your Own: Write the following ratios in lowest terms.  

\[
\begin{align*}
\frac{12}{60} &= \frac{1}{5} \\
\frac{15}{60} &= \frac{1}{4} \\
\frac{8}{24} &= \frac{1}{3} \\
\frac{6}{54} &= \frac{1}{9}
\end{align*}
\]

Solving Rate Problems

There are two methods to solve rate problems:

- With Algebra
- Without Algebra

Method 1: With Algebra

We can solve rate problems algebraically by using the method of proportions.

**Example:** Brandon runs 8 miles in 40 minutes. If he continues to run at the same speed, how far will he run in 50 minutes?

- Find the rate (in lowest terms): \(\frac{8 \text{ mi}}{40 \text{ min}} = \frac{1 \text{ mi}}{5 \text{ min}}\)

  Brandon's rate is \(1\) mile every \(5\) minutes.

- Set up a proportion and solve algebraically:

\[
\begin{align*}
\frac{1}{5} &= \frac{x}{50} \\
5x &= 50 \\
5x &= 50 \\
x &= 10
\end{align*}
\]

He can run 10 miles in 50 minutes.
On Your Own: Solve using the Algebra/Proportion Method

1. Keith biked 45 miles in 3 hours. If he continues to bike at that same speed, how many miles will he cover in 8 hours?

- Find the rate (in lowest terms):

\[
\frac{45}{3} = \frac{x}{8} \quad \text{Miles}
\]

- Set up a proportion and solve:

\[
\frac{45}{3} = \frac{x}{8}
\]

\[
3x = 360
\]

\[
x = 120
\]

2. Seth biked 60 miles in 4 hours. If he continues to bike at that same speed, how long will it take him to travel 90 miles?

- Find the rate (in lowest terms):

\[
\frac{60}{4} = \frac{90}{x} \quad \text{Miles}
\]

- Set up a proportion and solve:

\[
\frac{60}{4} = \frac{90}{x}
\]

\[
60x = 360
\]

\[
x = 6
\]
**Example:** Brandon runs 8 miles in 40 minutes. If he continues to run at the same speed, how far will he run in 50 minutes?

- **Find the rate** (by reducing the given ratio to lowest terms:

\[
\frac{8}{40} = \frac{1}{5} \quad \text{Rate: 1 mile per 5 minutes}
\]

- **Multiply** the rate (in fraction form) by the quantity we are given in the problem (50 minutes).

\[
\frac{1 \text{ mi}}{5 \text{ min}} \cdot 50 \text{ min} = \frac{1 \text{ mi}}{1 \text{ min}} \cdot \frac{10}{1} \text{ min} \quad \leftarrow \text{Cancel out common units and factors!}
\]

Solve: \(1 \text{ mi} \cdot 10 = 10\)

**Answer:** Brandon will run 10 miles in 50 minutes.

Another way to think about it is **intuitively**. If Brandon runs 1 mile in 5 minutes, then in 50 minutes he can run **10 times as far**:

\[1 \text{ mile} \cdot 10 = 10 \text{ miles}\]
On Your Own: Solve without algebra.  

1. Amy drives 90 miles in 1½ hours. If she continues driving at the same speed, how long will it take her to drive 270 miles?

Find the rate first and then apply it:

\[
\begin{align*}
90 \text{ miles} \text{ in } 1.5 \text{ hours} &= \frac{90 \text{ miles}}{1.5 \text{ hour}} = \frac{60 \text{ miles}}{1 \text{ hour}} \\
\text{Rate is } 60 \text{ mph} \\
\text{Apply rate to problem:} \\
\frac{1 \text{ hour}}{60 \text{ miles}} \cdot \frac{270 \text{ miles}}{9 \text{ hours}} &= \frac{9 \text{ hours}}{2} = 4.5 \text{ hours}
\end{align*}
\]

2. Irma ran 18 miles at the speed of 4½ miles per hour. How long did it take her to run that distance?

Rate: \(4.5 \text{ mph} \rightarrow 4.5 \text{ miles} : 1 \text{ hour} \rightarrow \frac{4.5 \text{ miles}}{1 \text{ hour}}\)

\[
18 \text{ miles} \cdot \frac{1 \text{ hour}}{4.5 \text{ miles}} = 18 \text{ hours} = 4 \text{ hours}
\]

Point out to students that \(18 \div 4.5\) can be more easily solved by moving the decimal point over for each number (i.e. multiplying both by 10) so that we get \(180 \div 45\).
Comparing Rates

On the CAHSEE, you may be asked to compare rates.

Example: Tim ran 150 meters in 25 seconds, and Evan ran 90 meters in 15 seconds. Based on these rates, which statement is true?

A. Tim’s average speed was 4 meters per second faster than Evan’s average speed.
B. Tim’s average speed was 2.4 meters per second faster than Evan’s average speed.
C. Tim’s average speed was 2 meters per second faster than Evan’s average speed.
D. Tim’s average speed was equal to Evan’s average speed.

To solve this problem, we need to find the rate in reduced form for each runner. Once we have both rates in reduced form, we can compare them:

Tim’s rate: \[\frac{150 \text{ meters}}{25 \text{ seconds}} = \frac{150}{25} = 6 \text{ meters per second}\]
Rate for Tim is \(6 : 1\)

Evan’s rate: \[\frac{90 \text{ meters}}{15 \text{ seconds}} = \frac{90}{15} = 6 \text{ meters per second}\]
Rate for Evan is \(6 : 1\)

Their rates are equal. The answer is D.

On Your Own: Alicia ran 15 miles in 3 hours. Dan ran 20 miles in 5 hours. Which of them ran the faster?

\[\frac{15 \text{ miles}}{3 \text{ hours}} = 5 \text{ miles per hour}\]

\[\frac{20 \text{ miles}}{5 \text{ hours}} = 4 \text{ miles per hour}\]

Alicia ran faster than Dan.
On the CAHSEE, you may be asked to order rates or speeds:

**Example:** Order the following speeds from fastest to slowest:

<table>
<thead>
<tr>
<th>Speed</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>360 ft/min</td>
<td>No change necessary: expressed in ft/min</td>
</tr>
<tr>
<td>35 yd/min</td>
<td>We need to convert from yards per minute to feet per minute. We start with the conversion factor:</td>
</tr>
<tr>
<td>24 ft/hr</td>
<td></td>
</tr>
</tbody>
</table>

Conversion Factor: \[ \frac{3 \text{ feet}}{1 \text{ yard}} \]

We can now multiply the rate (35 yd/min) by the conversion factor (expressed in fraction form) and cancel out common units:

\[
\frac{35 \text{ yards}}{1 \text{ minute}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = \frac{35 \cdot 3 \text{ feet}}{1 \text{ minute} \cdot 1 \text{ yard}} = 105 \text{ feet per minute}
\]
Now let's look at the last speed given: 24 ft/hr

1.3

- 24 ft/hr → To convert this rate to minutes, we need to multiply \( \frac{24 \text{ ft}}{1 \text{ hr}} \) by the conversion factor: 1 hour = 60 minutes

We can express this conversion factor as a fraction equal to 1:

\[
\frac{1 \text{ hour}}{60 \text{ minutes}}
\]

Note: We want the units of hours in the numerator because the rate is given with hours in the denominator: \( \frac{24 \text{ ft}}{1 \text{ hr}} \)

\[
24 \text{ feet} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{24 \text{ feet}}{60 \text{ minutes}} = \frac{2 \text{ feet}}{5 \text{ minutes}} = 0.4 \text{ ft per min}
\]

The last step is to order these speeds, from fastest to slowest. Be sure to write them in their original form.

<table>
<thead>
<tr>
<th>360 ft/min</th>
<th>35 yd/min (105 ft/min)</th>
<th>24 ft/hr (0.4 ft/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fastest</td>
<td>middle</td>
<td>slowest</td>
</tr>
</tbody>
</table>
Unit Quiz: The following questions appeared on the CAHSEE.

1. Beverly ran six miles at the speed of four miles per hour. How long did it take her to run that distance?
   - A. $\frac{2}{3}$ hr
   - B. $1 \frac{1}{2}$ hrs
   - C. 4 hrs
   - D. 6 hrs

2. Sixty miles per hour is the same rate as which of the following?
   - A. 1 mile per minute
   - B. 1 mile per second
   - C. 6 miles per minute
   - D. 360 miles per second

3. Order the following three speeds from fastest to slowest: 3,100 yd/hr, 160 ft/min, 9,200 ft/hr.
   - A. 9,200 ft/hr. 3,100 yd/hr 160 ft/min
   - B. 9,200 ft/hr 160 ft/min 3,100 yd/hr
   - C. 160 ft/min 9,200 ft/hr. 3,100 yd/hr
   - D. **160 ft/min 3,100 yd/hr 9,200 ft/hr.**
Introduction: Polygons & Quadrilaterals

A polygon is a **simple closed plane figure** made up of **three or more line segments**, such as a triangle. A **quadrilateral** is a polygon with **four sides**, such as a square. Examples of **polygons** are found below:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Trapezoid" /></td>
<td>Trapezoid</td>
<td>A <strong>quadrilateral</strong> with <strong>one pair of parallel sides</strong></td>
</tr>
</tbody>
</table>
| ![Parallelogram](image) | Parallelogram | A **quadrilateral** with **two pairs of parallel sides**  
**Note:** Sides with the same # of dashes are the same length. |
| ![Rhombus](image) | Rhombus | A **parallelogram** with **four equal sides** |
| ![Rectangle](image) | Rectangle | A **parallelogram** with **four right angles**  
**Note:** Sides with the same # of dashes are the same length. |
| ![Square](image) | Square | A **rectangle** with **four equal sides** and **four right angles** |
| ![Triangle](image) | Triangle | A **polygon** with **three sides** and **three angles** |
On the CAHSEE, you will be asked to find the **perimeter** (the distance around the outside of an object or shape) and **area** (the size of the inside of an object or shape) of both **polygons** and **circles**.

**Polygons** are flat, closed figures with three or more straight sides. Common polygons are **squares**, **rectangles**, **parallelograms**, and **triangles**. See page 34 for illustrations of polygons.

**Note:** Circles are not polygons: while they are flat, they do not have straight lines; rather, they are perfectly round.

**A. Perimeter ——— Add**

**Perimeter** is the distance around the outside of a polygon. To find the perimeter of a polygon, just add the measures of all of its sides; the sum is its perimeter.

**Note:** Perimeter is a linear, one-dimensional measurement and is used for polygons only. The term we use to describe the distance around a circle is "circumference," not perimeter.

One situation where perimeter would come in handy would be when framing a piece of fabric you want to hang on the wall. You would need to determine the perimeter of the fabric in order to buy the right size frame.

Think of two other situations where you would need to know the perimeter. (Possible answers below)

- To determine the number of yards of wood needed to build a fence
• To determine the amount of wallpaper trim needed to go around a room
**Example of Perimeter Problem**  

2.1

Find the perimeter of the rectangle below.

![Rectangle with dimensions](image)

**Note:** Although 3 and 3.5 are shown only once in the figure, we need to count them twice, since the sides opposite are of equal measure:

\[
3 + 3 + \underbrace{3.5 + 3.5}_{6 + 7} = 13 \text{ inches}
\]

**On Your Own:**

1. Find the perimeter of the square below.

![Square with dimensions](image)

\[
4 + 4 + 4 + 4 = 16 \text{ feet} \quad \text{Perimeter} = 16 \text{ feet}
\]
2. A rug measures 5 feet by 2 feet. Find the perimeter.

\[ 5 + 5 + 2 + 2 = 14 \text{ feet} \]

3. Find the perimeter of the equilateral triangle below.

**Note:** In an equilateral triangle, all three sides are equal.

\[ 5 + 5 + 5 = 15 \text{ inches} \]
4. Find the perimeter.

\[ \text{Perimeter} = 44 \text{ cm} \]

5. A wall painting measures 12' by 12'. Find the perimeter.

\[ \text{Perimeter: 48 feet} \]
While \textit{perimeter} measures the \textit{outside} of a figure, \textit{area} measures its \textit{inside}. The area of a figure is the number of \textit{square units} within that figure. It is a \textit{two-dimensional measurement}.

When would you ever need to measure area? List three situations below that would require the measure of area:

- To determine how much paint to buy for your living room
- To determine how much fertilizer to buy for your entire lawn
- To determine how much tile is needed to cover your kitchen
- To determine the amount of carpet to buy for a room

Area is always measured in \textit{square units} (because when units are multiplied by themselves, we get square units):

- square inches (in²)
- square feet (ft²)
- square meters (m²)
- square centimeters (cm²)

Let's examine various types of two-dimensional figures.

\textbf{i. Rectangles}

To find the area of a \textit{rectangle}, \textit{multiply} the \textit{base} by the \textit{height}.

\[
A = bh
\]

\textbf{Note:} Sometimes the dimensions are given as \textit{length and width}, rather than height and base.
Example: Find the area of the rectangle below.

Area = 3 in \times 3.5\text{ in} = 10.5 \text{ square inches}

On Your Own:

Find the area of the figure below:

Area = 2' \times 5' = 10 \text{ ft}^2
Practice

1. What is the area of the rectangle below?

\[
\text{Area} = \text{length} \times \text{width} = 4\text{cm} \times 8\text{cm} = 32\text{ cm}^2
\]

2. What is the area of the rectangle below?

\[
\text{Area} = 10\text{ in}^2
\]

3. What is the area of the rectangle in square units?

\[
\text{Area} = 6\text{ units}^2
\]

4. A rectangle has a height of 5 feet and a base of 12 feet. Find the area in square inches.

\[
\begin{array}{c|c}
\text{Height} & \text{Base} \\
5\text{ ft} & 12\text{ ft} \\
\end{array}
\]

\[
5\text{ ft} = 60\text{ in} \\
12\text{ ft} = 144\text{ in}
\]

\[
60\text{ in} \times 144\text{ in} = 8640\text{ in}^2
\]

Tutors: For hands-on exploration of area, use geoboards. (See the Tutor Supplement for Geoboard Activities.)
5. Find the area of the wall hanging below:

\[ \text{Area} = 90 \text{ ft}^2 \]

6. Find the area of the wall hanging below.

\[ \text{Area} = 19.25 \text{ ft}^2 \]
ii. **Squares**  

To find the area of a square, we apply the formula used for a rectangle: \( A = bh \).

**Remember:** In a square, all four sides are equal. Therefore we can multiply one side by itself.

\[
\text{Area of a Square} = bh \text{ (or } s^2)\]

**Example:** A square of fabric is shown below. Find the area.

\[
\text{Area} = (5)^2 = 25 \text{ square in.}
\]
On Your Own: 2.1 & 2.4

1. A hand-painted wall hanging measures 12' by 12'. Find the area.

\[
\text{Area} = 12 \times 12 = 12^2 = 144 \text{ ft}^2
\]

2. Find the area of a square with a length of 8". 

\[
\text{Area} = 64 \text{ in}^2
\]

3. Find the area of a square whose base is 11'. 

\[
\text{Area} = 121 \text{ ft}^2
\]

4. Each side of square A was multiplied by 3 to make square B.

\[
\begin{array}{c}
A \\
2 \\
\hline
B
\end{array}
\]

How does the area of square B compare to the area of square A?

A. It is three times as large.
B. **It is nine times as large.**
C. It is six times as large.
D. It is four times as large.
5. The wall hanging below measures 10' by 10'. Find the **perimeter** and the **area** of the hanging.

**Perimeter:** 40 ft

**Area:** 100 ft²
iii. **Triangles**

A **triangle** is a **polygon** with **three sides** and **three angles** (measured in degrees: $n^\circ$). The **sum** of all the angles is **180°**.

There are four different types of triangles that you should know:

1. A **scalene triangle** has **three sides** of **different lengths** and **three differentangles**.

![Diagram of a scalene triangle]

2. An **isosceles triangle** has **two congruent** sides (same length and shape) and **two similar angles**. (Note: Sides of the same length are shown with a dash.)

![Diagram of an isosceles triangle]

3. In an **equilateral triangle**, all **three sides and angles** are **equal**.

![Diagram of an equilateral triangle]

**Note:** Since the **sum** of all **angles** in a triangle is equal to **180°**, each of the three angles in an equilateral triangle is equal to $1/3$ of $180^\circ$, or $60^\circ$.

4. A **right triangle** has a **right angle** (an angle that measures $90^\circ$).

![Diagram of a right triangle]

**Note:** The two sides that form the $90^\circ$ angle are **perpendicular**.
Area of a Triangle

On the CAHSEE, you may be asked to find the area of a triangle. Notice that a triangle is half of a square:

Since the area of a square is base \( \times \) height, the area of a triangle is half of the area for a square:

\[
\text{Area of a Triangle} = \frac{1}{2} bh
\]

Note: Base (b) and height (h) are the horizontal and vertical sides of the triangle, respectively. The diagonal line is called the hypotenuse and is not used to find the area of the triangle.

Example: Find the area of the right triangle below.

\[
A = \frac{1}{2} bh
\]

\[
A = \frac{1}{2} (8)(6) = 24 \text{ square inches}
\]
On Your Own: 2.1

1. Find the area of the right triangle below.

\[ A = \frac{1}{2} \times \text{base} \times \text{height} \]

\[ A = \frac{1}{2} \times 9 \times 10 = 45 \text{ units}^2 \]

2. Find the area of the right triangle below.

\[ A = \frac{1}{2} \times 4 \times 3 = 6 \text{ units}^2 \]

Note: The diagonal line is the hypotenuse. Use only the base and height to solve this problem.

3. Find the area of the right triangle below.

\[ A = \frac{1}{2} \times 9 \times 3 = 13 \frac{1}{2} \text{ cm}^2 \]
4. In the **equilateral triangle** below, the **height** is **2 units** and the **base** is **4 units**. What is the area of the equilateral triangle?

\[ A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 2 = 4 \text{ units}^2 \]

**Note:** The height in an equilateral triangle is the same as it would be in a right triangle. Just look at one of the two right triangles in the larger equilateral triangle, and you will find the height:

5. Find the area of the **equilateral** triangle below:

Now find the area of one of the right triangles. (**Note:** We use the **same height** but **half of the base**.)

**Note:** Since each right triangle is equal to one-half of the equilateral triangle, we could just divide the area of the equilateral triangle by two.
iv. Circles

A circle is a flat, perfectly round figure. In order to find the area of a circle, we need to know the measure of either its diameter or radius:

The diameter is the distance across a circle through its center. The radius is half of the diameter.

\[
\text{diameter} = 2 \times \text{radius}
\]

**Note:** "Radii" is the plural of "radius."

To find the area of a circle, just take the square of the radius \( r \) and then multiply it by the number \( \pi \) (pi), which rounds to 3.14.

**Area of a Circle:**

\[
A = \pi r^2
\]

\[
r = \text{radius}
\]

\[
\pi \approx 3.14
\]

**Example:** Find the area of the circle below.

\[
A = \pi \cdot r^2
\]

\[
A = \pi \cdot (4 \text{ cm})^2
\]

\[
A = 3.14 \cdot 16 \text{ cm}^2 = 50.24 \text{ cm}^2
\]

**Note:** On the CAHSEE, answer choices may be given in terms of \( \pi \). In these problems, you will not need to multiply by 3.14. Just solve for \( r^2 \) and write \( \pi \) beside the answer.

For the example above, write the area in terms of \( \pi \): \( A = 16\pi \text{ cm}^2 \)
Example: Find the area of the circle below.

Note that in this example, we are given the diameter, not the radius. To find the area, we need to first figure out the radius.

We know that the radius is half of the diameter, so if the diameter is 24 mm, the radius is 12 mm.

Now we can continue with our calculations:

\[ A = \pi r^2 \]
\[ A = (12)^2 \pi = 144\pi \text{ mm}^2 \]

On Your Own: Each of the following measures corresponds to the diameter of a circle. Find the radius of each.

1. Diameter = 100 cm  \hspace{1cm} \text{Radius} = 50 \text{ cm}
2. Diameter = 15 mm  \hspace{1cm} \text{Radius} = 7\frac{1}{2} \text{ mm}
3. Diameter = 16"  \hspace{1cm} \text{Radius} = 8"
4. Diameter = 25 cm  \hspace{1cm} \text{Radius} = 12\frac{1}{2} \text{ cm}
5. Diameter = 22 mm  \hspace{1cm} \text{Radius} = 11 \text{ mm}
Practice: 2.1

1. Find the area of the circle shown below. Express in terms of $\pi$.

![Circle with radius 2 cm]

\[ A = \pi (2)^2 = 4\pi \text{ cm}^2 \]

2. Find the area of the circle shown below. Express in terms of $\pi$.

![Circle with radius 5 cm]

\[ A = \pi (5)^2 = 25\pi \text{ cm}^2 \]

3. Find the area of the smaller circle. Give your answer in terms of $\pi$.

![Circle with radius 2 inches and 7 inches]

\[ A = 7\pi \text{ in}^2 = \pi \text{ in}^2 \]
4. Find the area of a circle with a radius of 3 inches.

\[ A = 9\pi \text{ in}^2 \]

5. Find the area of a circle whose diameter is 8cm.

\[ A = 16\pi \text{ cm}^2 \]

6. In the circle below, \( r \) is equal to 9 cm. Find the area of the circle.

\[ A = 81\pi \text{ cm}^2 \]

7. Find the area of the circle below.

\[ A = 4\pi \text{ in}^2 \]

8. The diameter of the circle below is 6 cm. Find the area.

\[ A = 9\pi \text{ cm}^2 \]
The circumference of a circle is the **distance around** the circle.  
*(Note: It is equivalent to the *perimeter* of polygons. Circumference is given in *units, not square units*)*

In the picture above, the curved line marked $C$ represents the **circumference** of the circle.

To find the circumference of a circle, apply the formula:

$$C = \pi d \text{ or } C = 2\pi r$$

- $d =$ diameter
- $r =$ radius
- $\pi \approx 3.14$

**Example:** The circle below has a diameter of 4 inches. Find the circumference.

Since we are given the diameter, let's use the first formula: $C = \pi d$

$$C = (\pi)(4) = 12.56 \text{ inches}$$
On Your Own:  

2.1

1. The diameter of the circle below is 6 cm. Find the circumference.

   \[ C = \pi (6) = 18.84 \text{ cm} \]

2. Find the circumference of the circle below.

   \[ C = 2\pi (4) = 8\pi \text{ cm} = 25.12 \text{ cm} \]

3. Find the circumference of the circle shown below.

   \[ C = 2(\pi)(2) = 12.56 \text{ cm} \]

4. Find the area of the circle shown below.

   \[ C = \pi (10) = 31.4 \text{ cm} \]
Unit Quiz: The following questions appeared on the CAHSEE.

1. What is the area of the triangle shown below?

\[ \text{Area} = \frac{1}{2} \times 	ext{base} \times 	ext{height} = \frac{1}{2} 	imes 15 \times 11 = 82.5 \text{ square units} \]

A. 44 square units  
B. 60 square units  
C. 88 square units  
D. 120 square units.

2. Louis calculated the area of the circle below and got an answer of 50.769 cm\(^2\). He knew his answer was wrong because the correct answer should be about:

\[ \text{Area} = \pi r^2 \]

A. \( 4 \times 4 \times 4 = 64 \)  
B. \( 3 \times 3 \times 40 = 360 \)  
C. \( 31 \times 4 \times 4 = 496 \)  
D. \( 3 \times 40 \times 40 = 4800 \)
3. The two circles shown below have radii of 3 cm and 6 cm.

What is Circumference of Circle x?
Circumference of Circle y

A. \( \frac{1}{4} \)
B. \( \frac{1}{2} \)
C. \( \frac{\pi}{4} \)
D. \( \frac{\pi}{2} \)

4. The width of the rectangle shown below is 6 inches (in.). The length is 2 feet (ft).

What is the area of the rectangle in square inches?
A. 12
B. 16
C. 60
D. 144
5. In the figure below, the radius of the inscribed circle is 6 inches (in). What is the perimeter of square ABCD?

![Inscribed Circle and Square](image)

A. $12\pi$ in.
B. $36\pi$ in.
C. 24 in.
D. 48 in.

6. The points (1, 1), (2, 3), (4, 3), and (5, 1) are the vertices of a polygon. What type of polygon is formed by these points?

A. Triangle
B. Trapezoid
C. Parallelogram
D. Pentagon

**Solution:**

![Polygon on Graph Paper](image)
Unit 5: Three-Dimensional Measures

On the CAHSEE, you may be asked to identify common solids:

<table>
<thead>
<tr>
<th>Solid</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>Three-dimensional solid whose bases (faces) are squares</td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td>Three-dimensional solid whose bases are rectangles</td>
</tr>
<tr>
<td>Cone</td>
<td>Three-dimensional triangular solid whose base is a circle</td>
</tr>
<tr>
<td>Pyramid</td>
<td>Three-dimensional triangular solid whose base is a rectangle</td>
</tr>
<tr>
<td>Cylinder</td>
<td>Three-dimensional rectangular solid with two congruent bases that are circles</td>
</tr>
<tr>
<td>Sphere</td>
<td>Three-dimensional circular solid with all points the same distance from the center</td>
</tr>
</tbody>
</table>
Measures of Solid Figures 2.1 & 2.3

On the CAHSEE, you will be asked to find the volume and surface area of solid figures. Solid figures are three-dimensional. Unlike two-dimensional figures, which are flat, three-dimensional figures have depth.

A. Volume

**Volume** is a measure of how many cubic units (units$^3$) it takes to fill a solid figure. It is found by multiplying all of the dimensions of the figure.

**Note:** Volume is measured in cubic units.

The formulas for volume vary, depending on the type of three-dimensional solid, or prism.

A **prism** is a three-dimensional solid.

Let's look at the various types of prisms that may appear on the CAHSEE.

i. Rectangular Prisms

A rectangular prism is a three-dimensional solid that has bases that are rectangles.

Example
Volume of a Rectangular Prism

As we mentioned before, in a rectangular prism, the bases are rectangles. To find the volume of a rectangular prism, we multiply the area of the base of the solid figure by the height of the figure. Since the area of the base is length • width, the volume of a rectangular prism is length • width • height:

\[ \text{Volume of a Rectangular Prism} = lwh \]

\[ l = \text{length} \]
\[ w = \text{width} \]
\[ h = \text{height} \]

Example: Find the volume of the rectangular prism below.

\[ V = 6 \cdot 3 \cdot 5 = 90 \]

Note: For mental math, it is easier to regroup the numbers as follows: \((6 \cdot 5) \cdot 3 = 30 \cdot 3 = 90\)
\((30 \times 3 \text{ is easier to calculate than } 18 \times 5 \text{ or } 15 \times 6.)\]
2.1 & 2.3

On Your Own:

1. Find the volume of the rectangular prism.

\[ V = (5)(3)(4) = 60 \text{ cubic units} \]

\[ \text{Mental Math: Easier to group } (5 \cdot 4) \cdot 3 \rightarrow 20 \cdot 3 = 60 \]

2. Find the volume of a rectangular prism with a length of 6 cm, a width of 3 cm, and a height of 8 cm.

\[ V = 144 \text{ cm}^3 \]

\[ \text{Mental Math: } (8 \cdot 3) \cdot 6 = 24 \cdot 6, \text{ which is equal to } 12 \cdot 12 \]
\[ \text{(half of one, double of the other)} \]
3. Find the volume of the rectangular prism below.

\[ V = 40 \text{ units}^3 \]

4. Find the volume of the rectangular solid:

\[ V = 24 \text{ cubic units} \]
ii. Cubes

A cube is just a variation of a rectangular prism, in which all of the bases are squares. Just as a square is a special type of rectangle in which all four sides are equal, a cube is a rectangular prism with six congruent bases that are squares.

If we unfolded the cube, this is what it would look like:

Volume of a Cube

To find the volume of a cube, you can apply the formula for finding the volume of a rectangular prism \((V = lwh)\) or, since all of the bases are equal, you can just cube one of the sides:

\[
Volume \ of \ a \ Cube \ = \ s^3
\]

Example: Find the volume, in cubic units, of the cube below.

\[
V = (5)^3 = 125 \ \text{units}^3
\]
2.1 & 2.3

**On Your Own:**

1. Find the volume of a cube whose length is 4 cm.

   \[ V = (4)^3 = 64 \text{ cm}^3 \]

2. Find the volume of the cube below.

   \[ V = 27 \text{ units}^3 \]

3. Find the volume of the cube below.

   \[ V = 1 \text{ unit}^3 \]
Enlarging Cubes

On the CAHSEE, you may be asked to find the volume of a cube that has been enlarged by a certain number or multiplied by a certain number. Look at the following example:

Example: If the sides of the cube below are multiplied by two, the volume of the new cube is ____.
A. Doubled
B. Multiplied by 3
C. Multiplied by 6
D. Multiplied by 8

To solve this problem, we need to compare the volume of the original cube with the volume of the new cube.

• Let's first find the volume of the original cube:

\[ V = 2^3 = 2 \times 2 \times 2 = 8 \]

• If we multiply all of the sides by two, each side of the enlarged cube will be 4 units (2 \times 2).

Continued on next page
Now let's find the volume of the new cube:

\[ V = 4^3 = 4 \cdot 4 \cdot 4 = 64 \]

Our last step is to compare the volumes of the two cubes:

Volume of 1st Cube = 8 \hspace{1cm} Volume of 2nd cube = 64

**Answer:** The volume of the new cube (64) is 8 times greater than that of the first cube (8). Therefore, our answer is **Choice D**.

**On Your Own:**

1. The sides of the cube below are multiplied by 4. Find the volume of the new cube.

   \[ 2 \times 2 \times 2 = 8 \]

   **Answer:** \( 8^3 = 512 \text{ m}^3 \)

2. The cube below is enlarged so that it holds twice as much as the original.

   Should we . . .
   
   a. **Double the height**?
   
   b. Double both the length and width?
   
   c. Double both the length and height?
   
   d. Double the length, the width and the height.
iii. **Cylinders**  

A cylinder is a solid in which the bases are circles and the other surface is a rectangle wrapped around the circle. (Note: Most drink cans are cylinders.)

![Diagram of a cylinder](image)

**Volume of a Cylinder** = \( r^2 \pi h \)

- \( r \) = radius
- \( h \) = height
- \( \pi \approx 3.14 \)

**Example:** Find the volume of the cylinder below.

![Diagram of a cylinder with dimensions](image)

\[
V = r^2 \pi h \\
V = (4)^2(10) \pi = 160 \pi \text{ in}^3
\]

**Note:** It is a good idea to learn the formulas for the volume of a cube, rectangular prism, and cylinder, although, in the past, these formulas have been provided on the CAHSEE.
On Your Own: 2.1 & 2.3

1. Find the volume of the cylinder below.

\[ V = r^2 \pi h \]

\[ V = (3^2)(6)\pi = 54\pi \text{ in}^3 \]

2. Find the volume of the cylinder below.

\[ V = 45\pi \text{ in}^3 \]

3. Find the volume of the cylinder below.

\[ V = 441\pi \text{ mm}^3 \]
B. Surface Area  

Do you remember that **volume** is the measure of how many **cubic units** it takes to **fill a solid figure**? Well, **surface area** is the measure of how many **square units** it takes to **cover** a solid.

To find the surface area of a solid . . .

• First, find the **area** of each **face** (each **two-dimensional** plane surface):

<table>
<thead>
<tr>
<th>Area of Each Face:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length • Width (1st face)</td>
</tr>
<tr>
<td>Length • Height (2nd face)</td>
</tr>
<tr>
<td>Width • Height (3rd face)</td>
</tr>
<tr>
<td>(Repeat for faces 4 through 6)</td>
</tr>
</tbody>
</table>

• Second, **add** all of the **areas** together.

**Example:** Find the surface area of the rectangular prism below.

![Rectangular prism diagram]

**Note:** Since each face has an identical match on the other side of the prism; we can save time by **multiplying** each separate area by **2**:

\[
SA = 2(l \cdot w) + 2(l \cdot h) + 2(w \cdot h)
\]

\[
SA = 2(5 \cdot 2) + 2(5 \cdot 4) + 2(2 \cdot 4) = 20 + 40 + 16 = \boxed{76 \text{ units}^2}
\]

**Note:** Surface area is given in **square units** because area is always given in **square units**.
On Your Own

Find the surface area of the rectangular prism below.

\[ \text{SA} = 2(6 \cdot 3) + 2(6 \cdot 8) + 2(3 \cdot 8) = \]
\[ 36 + 96 + 48 = 180 \text{ units}^2 \]

Surface Area of Cubes

On the CAHSEE, you are more likely to be given a surface area problem that involves cubes. Like rectangles, cubes have six sides. However, all six sides of a cube are equal (since a cube is made of squares, which have equal sides).

As we have learned, to find the surface area of a solid, find the area of each face and add them all together. However, since a cube is made of 6 equal squares, we only need to find the area of one square \((s^2)\) and multiply this number by 6.
2.1 & 2.3

Example: Find the surface area of the square below.

\[
SA = 6s^2
\]

\[
SA = 6(2)^2 = 6(4) = 24 \text{ units}^2
\]

On Your Own

Find the surface area of the cube below.

\[
SA = 6(4)^2 = 6(16) = 96 \text{ units}^2
\]
Practice 2.1 & 2.3

1. Find the surface area of the cube below.

   \[ SA = 6(5^2) = 6(25) = 150 \text{ units}^2 \]

2. Find the surface area of the cube below.

   \[ SA = 6(3^2) = 6(9) = 54 \text{ units}^2 \]

3. Find the volume of the cube below.

   \[ SA = 6(1^2) = 6(1) = 6 \text{ units}^2 \]
2.1 & 2.3

Unit Quiz: The following questions appeared on the CAHSEE.

1. In the figure below, an edge of the larger cube is 3 times the edge of the smaller cube. What is the ratio of the surface area of the smaller cube to that of the larger cube?

A. 1 : 3
B. 1 : 9
C. 1 : 12
D. 1 : 27

2. A cereal manufacturer needs a box that can hold twice as much cereal as the box shown below.

Which of the following changes will result in the desired box? \((V = lwh)\)

A. Double the height only.
B. Double both the length and width.
C. Double both the length and height.
D. Double the length, width and height.
3. What is the volume of the cylinder shown below? \( (V = \pi r^2 h) \)

A. \(60\pi \text{ in.}^3\)
B. \(120\pi \text{ in.}^3\)
C. \(300\pi \text{ in.}^3\)
D. \(600\pi \text{ in.}^3\)

4. What is the volume of the shoebox shown below in cubic inches \( (\text{in}^3) \)?

A. 29
B. 75
C. 510
D. 675
5. Gina is painting the rectangular tool chest shown in the diagram below.

If Gina paints only the outside of the tool chest, what is the total surface area, in square inches (in.²), she will paint?

A. 368
B. 648
C. 1296
D. 2880
On the CAHSEE, you will be asked to find the area, perimeter or volume of irregular figures. Let's begin with perimeter.

A. Perimeter of Irregular Figures

**Example:** Find the perimeter of the figure below:

As you remember, **perimeter** is the **sum** of all of the sides. In this problem, there are **six** different sides to **add**.

**Note:** The lengths of some of the sides are not written. You must **figure them out**!

One way to make sure that you don’t forget a side in your calculations is to **put a dot** in the upper left corner, and then **go clockwise**, adding sides as you go around:

\[13 + 4 + 8 + 2 + 5 + 6 = 38 \text{ inches}\]
On Your Own:

1. Find the perimeter of the figure below:

2. Find the perimeter of the figure below:

\[
P = 13 + 7 + 2 + 4 + 6 + 4 + 5 + 7 = 48 \text{ units}
\]
Example: Find the area of the figure below:

To solve this problem, you must . . .

- Break the irregular figure down into two or more regular figures:

We can see that this figure consists of two smaller regular figures:

1. One square
2. One rectangle

To help us keep track of both figures, we number them 1 and 2.

- Find the area of each regular figure:

  \[
  \begin{align*}
  \text{Area of Figure}^1 & : 2 \times 2 = 4 \\
  \text{Area of Figure}^2 & : 8 \times 2 = 16
  \end{align*}
  \]

- Add the areas together:

  \[
  \text{Area of Figure}^1 + \text{Area of Figure}^2 = 4 + 16 = 20 \text{ units}^2
  \]
On the CAHSEE, you may also be asked to find the remaining area of a figure or the area of a shaded part of a figure:

**Example:** In the figure below, one small rectangle is cut from a larger rectangle. Find the area of the remaining figure.

![Diagram of a rectangle with a smaller rectangle cut out of it.](image)

- We can see that this irregular figure consists of two smaller regular figures:
  1. One small rectangle
  2. One large rectangle

  To keep track of both figures, number the smaller figure 1 and the larger figure 2.

- Find the area of each figure:

  **Area of Figure 1:** \(2 \times 4 = 8\)
  
  **Area of Figure 2:** \(8 \times 6 = 48\)

- Since we are asked to find the area of the remaining figure, we need to subtract the area of the smaller rectangle (Figure 1) from the area of the larger rectangle (Figure 2):
On Your Own

1. Find the area of the figure below.

![Figure](image)

\[
A = (5 \cdot 6) + (8 \cdot 4) = 30 + 32 = 62 \text{ ft}^2
\]

Or

\[
A = (13 \cdot 6) - (8 \cdot 2) = 78 - 16 = 62 \text{ ft}^2
\]

2. Find the area of the shaded part:

![Shaded Figure](image)

\[
\text{Area of Triangle} = \frac{1}{2} \cdot bh = \frac{1}{2} \cdot (210) = 105
\]

\[
\text{Area of Square} = 3^2 = 9
\]

\[
\text{Area of Triangle} - \text{Area of Square} = 105 - 9 = 96 \text{ ft}^2
\]
C. Volume of Irregular Figures

On the CAHSEE, you may be asked to find the **volume** of **irregular figures**:

**Example:** What is the volume of the figure below?

- The first step is to find the number of cubes. (Don’t forget to count the cubes that you can't see but know are there!) **12**

- Now apply the formula for finding the volume of 1 cube:

\[
V = s^3 \\
V = (5)^3 \\
V = 125
\]

- Finally, multiply this figure by the number of cubes:

\[
125 \times 12 = 1500 \text{ in}^3
\]
Unit Quiz: The following problems appeared on the CAHSEE.

1. A right triangle is removed from a rectangle as shown in the figure below. Find the area of the remaining part of the rectangle. (Area of a triangle = \( \frac{1}{2}bh \))

A. 40 in.²
B. **44 in.²**
C. 48 in.²
D. 52 in.²

2. A rectangular pool 42 feet by 68 feet is on a rectangular lot 105 feet by 236 feet. The rest of the lot is grass. Approximately how many square feet is grass?

A. 2,100
B. 2,800
C. **21,000**
D. 28,000
3. In the figure shown below, all the corners form right angles. What is the area of the figure in square units?

![Figure](image.png)

A. 67
B. 73
C. 78
D. 91

4. One-inch cubes are stacked as shown in the drawing below. What is the total surface area?

![Drawing](image.png)

A. 19 in.$^2$
B. 29 in.$^2$
C. 32 in.$^2$
D. 38 in.$^2$
5. The largest possible circle is to be cut from a 10-foot square board. What will be the approximate area, in square feet, of the remaining board (shaded region)? \(A = \pi r^2\) and \(\pi = 3.14\)

A. 20
B. 30
C. 50
D. 80

6. What is the area of the shaded region in the figure shown below? (Area of a triangle = \(\frac{1}{2}bh\))

A. 4 cm²
B. 6 cm²
C. 8 cm²
D. 16 cm²
7. The short stairway shown below is made up of solid concrete. The height and width of each step is 10 inches (in.). The length is 20 inches.

What is the volume, in cubic inches, of the concrete used to create this stairway?
A. 3000
B. 4000
C. 6000
D. 8000

8. In the figure below, every angle is a right angle.

What is the area, in square units, of the figure?
A. 96
B. 108
C. 120
D. 144
9. Mia found the area of this shape by dividing it into rectangles as shown.

Mia could use the same method to find the area of which of these shapes?

Answer: D
Unit 7: Congruence & Transformation  3.2 & 3.4

A. Congruence

Objects that are exactly the same size and shape are said to be congruent. This means that the corresponding sides (sides that match up) will have the same length and that the corresponding angles (angles that match up) will be equal in degrees.

Examples of Congruent Figures

![Examples of Congruent Figures](image)

Distinction between Similarity and Congruence

Figures that have the same shape are called similar figures. They may be different sizes; they may even be turned (rotated). However, as long as the shape remains the same, they are similar.

Examples of Similar Figures:

![Examples of Similar Figures](image)

Note: They have the same shape (so they are similar), but they do not have the same size (so they are not congruent)
CAHSEE Alert!  

Be careful! On the CAHSEE, you may be asked to identify congruent figures and presented with figures that have the same shape but not the same size.

Example: Which pair of figures is congruent? **Pair C**

![Example Figures](image)

Congruent Triangles

On the CAHSEE, you may be asked to determine whether two triangles are **congruent**.

**Triangles** are **congruent** when . . .

- they are the **same size**
- they are the **same shape**
- their **corresponding sides** are the **same length**
- their **corresponding angles** are **equal**

Examples of Congruent Triangles
Let's look closely at one pair of congruent triangles:

Triangle ABC is congruent to Triangle DEF: \( \triangle ABC \cong \triangle DEF \)

\( \cong \) means congruent

**Note:** Each set of dashes represents the length of a line segment, and each set of curved lines represents the measure of a given angle:

These triangles are congruent because...

- they are the same size
- they are the same shape
- their corresponding sides are the same length:
  - \( \overline{AB} \) (Line Segment AB) corresponds to \( \overline{DE} \)
  - \( \overline{AC} \) corresponds to \( \overline{DF} \)
  - \( \overline{BC} \) corresponds to \( \overline{EF} \)
- their angles are equal
  - \( \angle A \) (the angle made at point A) corresponds to \( \angle D \)
  - \( \angle B \) corresponds to \( \angle E \)
  - \( \angle C \) corresponds to \( \angle F \)
On Your Own:

1. Which shape is congruent to the shaded figure?

A

B

C

D

E
2. The following figures are congruent.

Their corresponding sides are the same length:
- Segment $\overline{AD}$ corresponds to Segment $\overline{FG}$
- Segment $\overline{AB}$ corresponds to Segment $\overline{FE}$
- Segment $\overline{BC}$ corresponds to Segment $\overline{EH}$
- Segment $\overline{CD}$ corresponds to Segment $\overline{GH}$

Their angles are equal:
- $\angle A$ corresponds to $\angle F$
- $\angle B$ corresponds to $\angle E$
- $\angle C$ corresponds to $\angle H$
- $\angle D$ corresponds to $\angle G$
3. The following figures are congruent.

Their corresponding sides are the same length:
- Segment $\overline{AD}$ corresponds to Segment $\overline{WZ}$
- Segment $\overline{AB}$ corresponds to Segment $\overline{WX}$
- Segment $\overline{BC}$ corresponds to Segment $\overline{XY}$
- Segment $\overline{CD}$ corresponds to Segment $\overline{YZ}$

Their angles are equal:
- $\angle A$ corresponds to $\angle W$
- $\angle B$ corresponds to $\angle X$
- $\angle C$ corresponds to $\angle Y$
- $\angle D$ corresponds to $\angle Z$

4. Are the figures below congruent? Explain.

Answer: No. They are similar but not congruent. The corresponding line segments have different lengths.
B. Geometric Transformations

A transformation is a change in the position of a geometric figure. The figure resulting from a transformation is congruent to the original figure. In transformations, we can slide the figure (a translation), flip the figure (a reflection), or turn the figure (a rotation). On the CAHSEE, you will be tested only on translations and reflections.

i. Translations

One of the simplest transformations is the translation. A translation slides a figure along a straight line in a certain direction.

Example: PQRS has been translated to HIJK.

Notice how the size and shape of the figure remains the same. All that changes is the position.

Translations are often called slides because a translation is the image of a geometric figure after a slide in a certain direction. To translate a figure, simply slide it to a new position, either horizontally (left or right) some distance and/or vertically (up or down).
Example: The triangle on the left side of the graph has been translated **3 units down** and **5 units to the right**.

![Graph showing translation](image)

We can see this by picking 1 point in the left triangle, and counting the number of units **up/down** and to the **left/right** that it takes to get to the same point in the triangle on the right.

Notice that the **size and shape** of the figure **remain the same**. All that changes is its **position**.
On Your Own: 3.2

1. The triangle on the left has been translated 2 units to the right.

2. Triangle A has been translated 4 units to the (right/left) and 8 units (up/down).
3. The graph of $y = x^2 - 4$ is shown below.

Which graph best represents the graph of this parabola that has been translated 2 units up?

Answer: D
ii. **Reflections**

To understand reflections, think about what happens when you look in the mirror. You see the exact image of yourself, but reversed.

What happens when you look in the mirror and raise your right hand?

*Your left hand is raised in the mirror reflection.*

Notice that in the mirror image, the left hand appears to be raised. What you're seeing in the mirror is a **reflection**. A reflection is the **image** of a geometric figure that has been **flipped** over a **line of reflection** (or a line of symmetry). We call reflections "flips" because the figure flips over the **line of reflection**.

**Line of Reflection**

A line of reflection is where the reflection takes place. This line is always **halfway** between the actual figure and its reflected image.

*Note:* The lines above are also lines of symmetry since, in each case, the line breaks the figure into two symmetric (equal and congruent) parts.

**Examples of a Reflection:**
On Your Own:

1. Does the following drawing represent a translation or reflection?

Answer: Translation

2. Does the following drawing represent a translation or reflection?

Answer: Reflection
3.2  

3. Does the following drawing represent a translation or reflection?

4. Does the following drawing represent a translation or reflection?

5. Does the following drawing represent a translation or reflection?
6. Which figure will result if the triangle is reflected across the y-axis?

A  
B  
C  
D  
E
7. Which type of transformation is shown below? **Translation**

![Diagram of translation](image)

8. Which type of transformation is shown below? **Reflection**

![Diagram of reflection](image)

9. Which type of transformation is shown below? **Reflection**

![Diagram of reflection](image)
Unit Quiz: The following questions appeared on the CAHSEE.

1. Which of the following triangles $R'S'T'$ is the image of triangle $RST$ that results from reflecting triangle $RST$ across the $y$-axis?

Answer B  
Standard 3.2
2. Which figure is congruent to the figure shown below?

Answer: B

Standard 3.4
3. The graph of rectangle $ABCD$ is shown below.

What is the area, in square units, of rectangle $ABCD$?

A. 6  
B. 10  
C. **12**  
D. 14  

*Standard 3.2*
4. In the diagram below, hexagon $LMNPQR$ is congruent to hexagon $STUVWX$.

Which side is the same length as $MN$?

A. NP  
B. TU  
C. UV  
D. WX  

Standard 3.4
In right triangles, there is a special relationship between the hypotenuse (the side opposite the right angle: the diagonal line) and the legs (the horizontal and vertical sides) of the triangle.

This relationship is expressed in the Pythagorean Theorem.

The Pythagorean Theorem: The square of the hypotenuse equals the sum of the squares of the two legs:

\[ a^2 + b^2 = c^2 \]

where...

- \(a\) and \(b\) are the two legs of the triangle
- \(c\) is the hypotenuse

If we know any two sides of a right triangle, we can find the third by plugging the known values into the formula.

Example: Given a right triangle with a hypotenuse of 5" and a leg of 3", find the dimensions of the second leg.

\[ a^2 + b^2 = c^2 \]

\[ (3)^2 + b^2 = (5)^2 \]

Plug in the values that you know.

\[ 9 + b^2 = 25 \]

Simplify: Square the known values.

\[ b^2 = 16 \]

Combine common terms: Subtract 9 from both sides.

\[ b = \sqrt{16} \]

Take the (positive) square root.

Find the value of \(b\): 4

The second leg has a length of 4"
**Example:** Find the height of the right triangle (formed by dividing the equilateral triangle in half).

\[ a^2 + b^2 = c^2 \]

**Note:** The base of the **equilateral triangle** is 16. What is the base of the right triangle? 8

\[ (8)^2 + b^2 = (10)^2 \]

\[ 64 + b^2 = 100 \]

\[ b^2 = 36 \]

\[ b = \sqrt{36} \]

\[ b = 6 \]

The height of the right triangle is 6".

**Note:** While a, b and c are the variables used in the Pythagorean Theorem, you may encounter other variables on the CAHSEE. For instance, the hypotenuse may be labeled x, the height may be labeled \( y \), the base may be labeled c. You must be able to recognize the base, the height, and the hypotenuse on your own:
Pythagorean Triples

There are three sets of Pythagorean triples that appear over and over in math problems. Knowing them will save you a lot of time.

- **1st** Set of Triples: 3, 4, 5 — The hypotenuse is the larger #

![Diagram of a right triangle with sides 3, 4, and 5]

- **2nd** Set of Triples: 5, 12, 13
- **3rd** Set of Triples: 8, 15, 17

Every set of multiples will also give the sides of a right triangle. If you memorize the first set of triples, you can easily find the multiples. Just keep multiplying by the same factor.

1st set of Triples and their Multiples

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

Fill in the blank:

![Diagram with a right triangle and side lengths]

Test the theorem with these numbers: $6^2 + 8^2 = 10^2$

Does the theorem hold true? **Yes** $(36 + 64 = 100)$
### 2nd Set of Triples and their Multiples

Complete the chart:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12</td>
<td>13</td>
<td></td>
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<td>10</td>
<td>24</td>
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</tr>
<tr>
<td>15</td>
<td>36</td>
<td>39</td>
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</table>

### 3rd Set of Triples and their Multiples

Complete the chart:

<table>
<thead>
<tr>
<th></th>
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<th>b</th>
<th>c</th>
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</tr>
<tr>
<td>24</td>
<td>45</td>
<td>51</td>
<td></td>
</tr>
</tbody>
</table>

### Learn for the CAHSEE:

The hypotenuse is always the larger #
On Your Own:  

1. Find the value of $b$.

![Triangle](image)

$b = 15''$

**Recognize the Pythagorean triples: 8, 15, 17**

2. Find the value of $a$.

![Triangle](image)

$a = 9$ units

**Recognize the Pythagorean triples: 9, 12, 15 (from 3, 4, 5)**

3. Find the value of $c$.

![Triangle](image)

c = 10''

**Recognize the Pythagorean triples: 6, 8, 10 (from 3, 4, 5)**
4. Find the value of $b$.

Recognize the Pythagorean triples: 12, 16, 20 (from 3, 4, 5)

\[ b = 16 \text{ units} \]

5. Find the value of $a$.

Recognize the Pythagorean triples: 5, 12, 13

\[ a = 5' \]

6. Find the value of $x$.

Recognize the Pythagorean triples: 9, 12, 15 (from 3, 4, 5)

Note to Tutors: Here, the hypotenuse is assigned the variable of $x$, rather than $c$. While $a$, $b$ and $c$ are the variables used in the Pythagorean Theorem, students may encounter any variable on the CAHSEE. They must be able to recognize the base, the height, and the hypotenuse on their own.
7. Find the value of $y$.

$$y = 3 \text{ mm}$$

(Pythagorean Triples: 3, 4, 5)

8. Two points in the x-y-plane have coordinates (1, 5) and (3, 1). The distance between them is equal to the square root of which number?

Solution:

$$a^2 + b^2 = c^2$$

$$16 + 4 = c^2$$

$$20 = c^2$$

$$\sqrt{20} = c$$

Answer: The distance between the coordinates is equal to the square root of 20.
Unit Quiz: The following problems appeared on the CAHSEE:

1. What is the value of $x$ in the triangle shown below?

   $x = 13$

2. The club members hiked 3 kilometers north and 4 kilometers east, but then went directly home as shown by the dotted line in the figure below. How far did they travel to get home?

   They traveled 5 km.

3. What is the value of $x$ in the right triangle shown below?

   $x = 12'$

Tutors: For an overall review of this strand, play Geometry Bingo (in the Tutor Supplement for this strand).