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UC Davis, School and University Partnerships

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Introduction to the CAHSEE

The CAHSEE stands for the California High School Exit Exam. The mathematics section of the CAHSEE consists of 80 multiple-choice questions that cover 53 standards across 6 strands. These strands include the following:

Number Sense (14 Questions)

Statistics, Data Analysis & Probability (12 Questions)

Algebra & Functions (17 Questions)

Measurement & Geometry (17 Questions)

Mathematical Reasoning (8 Questions)

Algebra 1 (12 Questions)

What is CAHSEE on Target?

CAHSEE on Target is a tutoring course specifically designed for the California High School Exit Exam (CAHSEE). The goal of the program is to pinpoint each student’s areas of weakness and to then address those weaknesses through classroom and small group instruction, concentrated review, computer tutorials and challenging games.

Each student will receive a separate workbook for each strand and will use these workbooks during their tutoring sessions. These workbooks will present and explain each concept covered on the CAHSEE, and introduce new or alternative approaches to solving math problems.

What is Algebra & Functions?

Algebra & Functions is equivalent to pre-algebra; it consists of all of the foundational skills needed to succeed in algebra, including translating phrases into algebraic expressions, simplifying and evaluating algebraic expressions, solving linear equations and inequalities, and graphing linear equations. These topics are presented as separate units in this workbook.
Unit I: Translation of Problems into Algebra

Algebra is a language of its own. Algebra readiness involves learning this language and using it to translate simple English phrases and statements into algebraic expressions and equations (or inequalities). We will begin this unit with an introduction to the language of algebra.

A. The Language of Algebra

Read the following word problem.

Myra earns $15 each week in allowance. If she saves all of her allowance each week, how many weeks will it take her to earn enough to buy the new dress, which costs $90?

This problem can be solved using algebra. What we need to solve for (the number of weeks needed to save $90) is referred to in algebra as “the unknown variable” and is represented by a letter, usually $x$.

Let $x$ equal the number of weeks it will take Myra to buy the dress:

\[ x = \text{# weeks} \]

Once we determine our unknown variable, we can “translate” the problem, using the language of algebra:

We are told that Myra saves $15 each week. Each usually means “multiply.” So $15 \times$ the number of weeks ($x$) gives us the total dollars earned:

15\(x\) means $15$ each week for $x$ number of weeks

We are also told that this amount (15\(x\)) must be equal to (=) $90.

We can now translate the problem as an algebraic equation:

\[ 15x = 90 \]

We will later learn to solve algebraic equations, such as the one above.
B. Translating from English to Math: Key Words

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
</tr>
<tr>
<td>more</td>
<td>fewer than</td>
<td>times</td>
<td>half ((\div 2))</td>
</tr>
<tr>
<td>more than</td>
<td>less than*</td>
<td>twice ((\cdot 2))</td>
<td>one-third ((\div 3))</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>each</td>
<td>one-fourth ((\div 4))</td>
</tr>
<tr>
<td>exceeds</td>
<td>diminished by</td>
<td>of</td>
<td>per</td>
</tr>
<tr>
<td>spends</td>
<td></td>
<td>percent</td>
<td>out of</td>
</tr>
</tbody>
</table>

* Be careful with the expression "less than."

**Example:** Two is three less than five: \(2 = 5 - 3\)

Notice that in the word expression, 3 comes before 5, while in the math equation, 5 comes before 3. The order is reversed.

**Note:** \(5 - 3 = 2\) but \(3 - 5 = -2\)

C. Unknown Variables

In algebra, an unknown or undefined number is referred to as an unknown variable, and is represented with a letter, such as \(x\).

**Example:** Five more than a number: \(5 + x\)
**Examples: Translating to Algebraic Expressions**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eight more than a number</td>
<td>8 + x</td>
</tr>
<tr>
<td>Eight less than a number</td>
<td>x – 8</td>
</tr>
<tr>
<td>Notice how this changed order!</td>
<td></td>
</tr>
<tr>
<td>Eight decreased by a number</td>
<td>8 – x</td>
</tr>
<tr>
<td>Compare with above example.</td>
<td></td>
</tr>
<tr>
<td>Eight less than four times a number</td>
<td>4x – 8</td>
</tr>
</tbody>
</table>

**On Your Own:** Translate the phrases below to algebraic expressions.

<table>
<thead>
<tr>
<th>Word Expression</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Six more than a number</td>
<td></td>
</tr>
<tr>
<td>2. Six less than a number</td>
<td></td>
</tr>
<tr>
<td>3. Six times a number</td>
<td></td>
</tr>
<tr>
<td>4. One-sixth of a number</td>
<td></td>
</tr>
<tr>
<td>5. A number increased by 6</td>
<td></td>
</tr>
<tr>
<td>6. A number decreased by 6</td>
<td></td>
</tr>
<tr>
<td>7. 6 decreased by a number</td>
<td></td>
</tr>
<tr>
<td>8. 6 increased by a number</td>
<td></td>
</tr>
</tbody>
</table>
Learn these inequality signs for the CAHSEE.

Remember: the **open** part of the symbol always **faces the larger quantity**. One way to remember this is to think of the inequality sign as the mouth of a number-eating alligator; when faced with a choice, the alligator will always prefer the **bigger number**:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>is less than</td>
<td>&lt;</td>
</tr>
<tr>
<td>is greater than</td>
<td>&gt;</td>
</tr>
<tr>
<td>is less than or equal to</td>
<td>≤</td>
</tr>
<tr>
<td>is no more than</td>
<td>≤</td>
</tr>
<tr>
<td>is greater than or equal to</td>
<td>≥</td>
</tr>
<tr>
<td>is at least</td>
<td>≥</td>
</tr>
<tr>
<td>equals</td>
<td>=</td>
</tr>
<tr>
<td>is equal to</td>
<td>=</td>
</tr>
<tr>
<td>is, are, was, were, gives</td>
<td>=</td>
</tr>
</tbody>
</table>

→ 999,999 is less than 1,000,000

→ 1,000,000 is greater than 999,999
**On Your Own:** Fill in the spaces in the right column with the correct equality or inequality sign.

<table>
<thead>
<tr>
<th>Description</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number is less than five.</td>
<td>( x &lt; 5 )</td>
</tr>
<tr>
<td>A number is greater than five.</td>
<td>( x &gt; 5 )</td>
</tr>
<tr>
<td>A number is less than or equal to five.</td>
<td>( x \leq 5 )</td>
</tr>
<tr>
<td>Five times a number is equal to twenty.</td>
<td>( 5x = 20 )</td>
</tr>
<tr>
<td>15 is less than 5 times a number</td>
<td>( 15 &lt; 5x )</td>
</tr>
<tr>
<td>5 times a number is greater than 15.</td>
<td>( 5x &gt; 15 )</td>
</tr>
<tr>
<td>4 less than twice a number is greater than 16.</td>
<td>( 2x - 4 &gt; 16 )</td>
</tr>
<tr>
<td>Half of a number is equal to 11.</td>
<td>( \frac{x}{2} = 11 )</td>
</tr>
<tr>
<td>One-third of a number equals 12.</td>
<td>( \frac{x}{3} = 12 )</td>
</tr>
<tr>
<td>One-third of a number is less than 12.</td>
<td>( \frac{x}{3} &lt; 12 )</td>
</tr>
<tr>
<td>The product of five and a number is greater than 35.</td>
<td>( 5x &gt; 35 )</td>
</tr>
</tbody>
</table>
### Practice: Translate the statements below to algebraic equations or inequalities.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Algebraic Equation/Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Three more than twice a number is equal to 15.</td>
<td></td>
</tr>
<tr>
<td>2. Six less than two times a number is greater than 18.</td>
<td></td>
</tr>
<tr>
<td>3. Six decreased by two times a number is at least 18.</td>
<td></td>
</tr>
<tr>
<td>4. Eight increased by twice a number is less than 20.</td>
<td></td>
</tr>
<tr>
<td>5. Three times a number is equal to three more than half of that number.</td>
<td></td>
</tr>
<tr>
<td>6. Multiply a number by four and subtract seven from the result. The answer is five.</td>
<td></td>
</tr>
<tr>
<td>7. Six less than twice a number equals 16.</td>
<td></td>
</tr>
<tr>
<td>8. Nine decreased by a number is four.</td>
<td></td>
</tr>
<tr>
<td>9. Four increased by half of a number is equal to six.</td>
<td></td>
</tr>
<tr>
<td>10. Two less than six times a number is eight.</td>
<td></td>
</tr>
</tbody>
</table>
Unit Quiz: The following problems appeared on the CAHSEE.

1. Ten less than a number, $x$, is greater than twice the number.
   A. $10 - x > 2x$
   B. $10 - x < 2x$
   C. $x - 10 > 2x$
   D. $x - 10 < 2x$

2. Divide a number by 5 and add 4 to the result. The answer is 9. Which of the following equations matches these statements?
   A) $4 = 9 + \frac{n}{5}$
   B) $\frac{n}{5} + 4 = 9$
   C) $\frac{5}{n} = 4$
   D) $\frac{n + 4}{5} = 9$

3. In a certain room, the number of chairs, $c$, is equal to 3 times the number of tables, $t$. Which equation matches the information?
   A. $3 \cdot c = t$
   B. $3 \cdot t = c$
   C. $3 \cdot c = 3 \cdot t$
   D. $c \cdot t = 3$
4. Which of the following inequalities represents the statement, “A number, \( x \), decreased by 13 is less than or equal to 39”?

A. \( 13 - x \geq 39 \)
B. \( 3 - x \leq 39 \)
C. \( x - 13 \leq 39 \)
D. \( x - 13 < 39 \)

5. A shopkeeper has \( x \) kilograms of tea in stock. He sells 15 kilograms and then receives a new shipment weighing \( 2y \) kilograms. Which expression represents the weight of the tea he has now?

A. \( x - 15 - 2y \)
B. \( x + 15 + 2y \)
C. \( x + 15 - 2y \)
D. \( x - 15 + 2y \)
Unit II: Simplifying and Evaluating Algebraic Expressions

You have learned to translate word problems into algebra; now you will learn to **simplify** and **evaluate** (solve) algebraic expressions. Since many of the expressions contain multiple mathematical operations, the first step is to learn the correct order to follow.

### A. Order of Operations

When you have more than one operation in a math problem, you must perform the operations in a certain order. This order is called the “Order of Operations.”

One way to memorize the correct order of operations is to use this catchy **mnemonic device**:

Please    Excuse    My Dear    Aunt Sally

↓↓ ↓↓ ↓↓ ↓↓
Parentheses  Exponents  Multiply or Divide*  Add or Subtract**

*Note: The operations of multiplication and division are equal in power. Whether you use one before the other depends on which operation appears first (going from left to right).

**Note: The operations of addition and subtraction are equal in power. Whether you use one before the other depends on which operation appears first (going from left to right).
Example: \((3^2 + 5) - 2 \cdot 5 = \) _____

This problem involves many mathematical operations, all of which must be performed in the correct order:

<table>
<thead>
<tr>
<th>Please</th>
<th>Excuse</th>
<th>My Dear</th>
<th>Aunt Sally</th>
</tr>
</thead>
</table>

**Steps:**

- According to the order or operations, we need to solve what’s in the parentheses first:
  \[ (3^2 + 5) \]

- But before we can add \(3^2\) and 5, we need to raise 3 to the second power (since exponents come before addition):
  \[ (3^2 + 5) = (9 + 5) = 14 \]

So the problem \((3^2 + 5) - 2 \cdot 5\) can be simplified as follows:

\[ 14 - 2 \cdot 5 \]

- According to the order of operations, multiplication comes before subtraction. **Note:** We multiply the numbers 2 and 5 because the multiplication sign comes between these two numbers.
  \[ 14 - (2 \cdot 5) = 14 - \_\_ \]

- The last step is to subtract.
  \[ \_\_ - \_\_ = \_\_ \]
Example: $10 - 2 \cdot 3 \div 2$

Follow the order of operations:

<table>
<thead>
<tr>
<th>Parentheses</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponents</td>
<td>None</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$2 \cdot 3 = ____$</td>
</tr>
<tr>
<td>Division</td>
<td>$___ \div 2 = ____$</td>
</tr>
<tr>
<td>Addition</td>
<td>None</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$10 - ____ = ____$</td>
</tr>
</tbody>
</table>

We can also look at the problem this way:

$$10 - 2 \cdot 3 \div 2 =$$
$$10 - (2 \cdot 3 \div 2) =$$
$$10 - (___ \div 2) =$$
$$10 - ____ = ____$$
On Your Own

Example: \((13 - 5) + 2 \cdot 5 = \) __________

Follow the order of operations:

<table>
<thead>
<tr>
<th>Parentheses</th>
<th>((13 - 5) = ) ___</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponents</td>
<td>None</td>
</tr>
<tr>
<td>Multiplication and/or Division</td>
<td>(2 \cdot 5 = ) ___</td>
</tr>
<tr>
<td>Addition and/or Subtraction</td>
<td>___ + ___ = ___</td>
</tr>
</tbody>
</table>

Answer: ___

Example: \(13 - (5 + 2) \cdot 5 = \) __________

Follow the order of operations:

<table>
<thead>
<tr>
<th>Parentheses</th>
<th>(___ + __) = ___</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponents</td>
<td>None</td>
</tr>
<tr>
<td>Multiplication and/or Division</td>
<td>___ (\cdot 5 = ) ___</td>
</tr>
<tr>
<td>Addition and/or Subtraction</td>
<td>(13 - 35 = ) ___</td>
</tr>
</tbody>
</table>

Answer: ___
Example: $3 \cdot (3 + 2) \div 5 = \underline{______}$

Follow the order of operations:

<table>
<thead>
<tr>
<th>Parentheses</th>
<th>Exponents</th>
<th>Multiplication and/or Division</th>
<th>Addition and/or Subtraction</th>
</tr>
</thead>
</table>

Answer: ___

Importance of the Order of Operations

It is important to follow the order of operations when solving problems that involve several mathematical operations. Otherwise, you may get the wrong answer.

Example: $5 + 4 \cdot 3 = \underline{______}$

According to the order of operations, multiplication comes before addition. Therefore, we should read the problem in the following way:

$5 + (4 \cdot 3) = \underline{______}$ What is your answer? ____

Let’s look at the problem again. What would you get if you added first and then multiplied? $(5 + 4) \cdot 3 = \underline{______}$

Note: These two answers are not the same. Failing to multiply first results in the wrong answer!
Fractions and the Order of Operations

Example: \( \frac{3 \cdot 10}{2 + 3} + 4 = \) ________

Note: A fraction bar serves as a division symbol. Any terms grouped in the numerator and/or denominator should be evaluated separately and treated as numbers within parentheses. First, evaluate the numerator; second, evaluate the denominator; third, divide:

\[
\frac{3 \cdot 10}{2 + 3} + 4 = \frac{30}{5} + 4 = 6 + 4 = 10
\]

On Your Own: Use the Order of Operations to solve.

\[
\frac{18 + 9}{15 - 6} + 11 = \frac{27}{9} + 11 = \_\_ \_ + 11 = \_\_\_ 
\]

\[
81 \div \frac{24 + 12}{13 - 9} = 81 \div \_\_ = 81 \div \_\_ = \_\_\_ 
\]

\[
3 \cdot \frac{18 + 3}{7} = \_\_\_ 
\]

\[
16 - \frac{12 \times 3}{6} = \_\_\_ 
\]
Absolute Value and the Order of Operations

The absolute value bars are treated as parentheses. Evaluate any expression between absolute value bars before performing any other operations.

**Example:** \(13 - |-18 + 6| = \) ______

Follow the order of operations:

| Parentheses          | \(|-18 + 6| = |-12| = 12\) Remember: The absolute value of a number is always positive. |
|----------------------|---------------------------------------------------------------------------------|
| Exponents            | None                                                                             |
| Multiplication and/or Division | None                                                                 |
| Addition and/or Subtraction | 13 - 12 = 1                                                                   |

**On Your Own:** Use the Order of Operations to solve.

\[
25 + |15 - 7| = 25 + |___| = 25 + ___ = ___
\]

\[
3^2 \cdot |8 - 15| = ___ \cdot |___| = 9 \cdot ___ = ___
\]

\[
56 ÷ |-8 + 15| = 56 ÷ |___| = 56 ÷ ___ = ___
\]
Practice: Use the order of operations to solve.

1. $5 + 8 \cdot 4 = \phantom{0}$
   - Which operation comes first? ________________
   - Solve: ________________

2. $(5 + 8) \cdot 4 = \phantom{0}$
   - Which operation comes first? ________________
   - Solve: ________________

3. $6^2 + 3 \cdot 2 = \phantom{0}$
   - Which operation comes first? ________________
   - Which operation comes second? ________________
   - Which operation comes third? ________________
   - Solve: ________________

4. $(6^2 + 3) \cdot 2$
   - Which operation comes first? ________________
   - Which operation comes second? ________________
   - Which operation comes third? ________________
   - Solve: ________________
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 5. | \[
\begin{align*}
14 + 4 &= 18 \\
48 ÷ 8 &= 6
\end{align*}
\]
Which operation comes first? ________________
Which operation comes second? ________________
Which operation comes third? ________________
Solve: ________________

| 6. | \[
13 + |11 - 21| ÷ 5
\]
Which operation comes first? ________________
Which operation comes second? ________________
Which operation comes third? ________________
Solve: ________________

| 7. | \[
\frac{22 + 13}{5} - 11
\]
Which operation comes first? ________________
Which operation comes second? ________________
Which operation comes third? ________________
Solve: ________________
**Quiz on Order of Operations:** Show all work.

1. \(3^2 \cdot 4^3 = \ldots\)

2. \(7 + 3 \cdot 2^4 \div 6 = \ldots\)

3. \(4^3 \div 2^3 \cdot 2 = \ldots\)

4. \(2(3 \cdot 4)^2 = \ldots\)

5. \((7 + 3) \cdot 2^4 \div 2 = \ldots\)

6. \(3^2 - 6 \cdot 2 = \ldots\)

7. \((3^2 - 6) \cdot 2 = \ldots\)

8. \((3 - 6)^2 \cdot 2 = \ldots\)

9. \(33 \cdot \frac{11 \cdot 4}{30 - 8} + 11 = \ldots\)

10. \(48 \div |19 - 23| \cdot 5 = \ldots\)
B. Simplifying Algebraic Expressions

The order of operations is also used with algebraic expressions (i.e. expressions that have one or more unknown variables). In this section, we will learn to simplify expressions with two monomials, expressions with one monomial and one binomial, and expressions with two binomials. Let’s begin by reviewing the vocabulary.

Vocabulary Review

**Monomial Expression:** An expression consisting of one term

Examples:

- $4x^2z^2$ ← One Term
- $18abc$ ← One Term
- $\frac{3}{ab}$ ← One Term

**Binomial Expression:** An expression consisting of two terms connected by the plus (+) or minus (-) sign.

Examples:

- $2x^2 + 3$ ← Two Terms
- $4 - x$ ← Two Terms
- $x + 5$ ← Two Terms
- $4x + 3$ ← Two Terms
- $4x^2 + 3x$ ← Two Terms
**Trinomial Expression**: An expression consisting of **three** terms connected by the plus (+) or minus (−) sign.

**Examples:**

\[2x^2 + 3x + 5 \quad \text{← Three Terms}\]

\[4x^2 + x - 2 \quad \text{← Three Terms}\]

\[x^2 - 5x - 18 \quad \text{← Three Terms}\]

**Note**: When writing algebraic expressions, it is customary to write the **variables** in order of decreasing powers/exponents. The **constant** (integer) comes last.

**Example**: \[3x^4 + 4x^3 - 2x^2 + 7x + 3\]

\[\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow\]

| Power of 4 | Power of 3 | Power of 2 | Power of 1 | Constant |

**Note**: The **coefficient** is the numeral preceding the variable.

**Example**: \[3x^4\]

\[\uparrow\]

**Coefficient is 3**
i. **Simplifying Expressions with Two Monomials**

To simplify algebraic expressions with two monomials, follow these steps:

A. **Multiply** the **coefficients** (numerals) preceding **both terms**
B. **Multiply** the **variables** of **both terms**
C. **Combine** the **coefficients and variables** into **one expression**

**Example: 2x \cdot 2y**

\[
\begin{array}{c}
\text{2} \\
\times \\
\text{2} \\
\hline \\
\text{4} \\
\end{array}
\]

\[
(2 \cdot 2) \cdot (x \cdot y) = 4 \cdot xy = 4xy
\]

**On Your Own**

1. \(3x \cdot 4x = \_\_\_\_\_

   A. Multiply coefficients: \(3 \cdot 4 = 12\)
   B. Multiply variables: \(x \cdot x = x^2\)
   C. Combine: \(12 \cdot x^2 = \_\_\_\_\_

2. \(-4a^3b^4 \cdot 3c^2d^5 = \_\_\_\_

   A. \_\_\_\_
   B. \_\_\_\_
   C. \_\_\_\_
3. \(-5mn \cdot -4\)

4. \(5n \cdot 4mn\)

5. \(-6x \cdot 2y^2\)

6. \(-4x^2 \cdot -6y^2\)

7. \(8a^{2}b \cdot -7ab^2\)

**Note:** In Unit 3, we will learn to multiply and divide more complex monomial expressions.
ii. **Simplifying Expressions with One Monomial and One Binomial**

Look at the following expression:

**Example: 3(4 + 5)**

This expression has one **monomial** (3) and one **binomial** (4 + 5). To **simplify**, we clear the parentheses. This means that we **multiply** each term in the binomial by the monomial. We can do this by using the **distributive law of multiplication**:

\[
3(4 + 5) = 3(4 + 5) \\
12 + 15 = 17
\]

**Distributive Law of Multiplication:**

For any three numbers a, b, and c,

\[
a(b + c) = (a \cdot b) + (a \cdot c) = ab + ac
\]

**Distributive Law of Multiplication Using Integers**

Let's see how the distributive law works when multiplying integers:

\[
3(4 + 2) = \\
(3 \cdot 4) + (3 \cdot 2) = \\
12 + 6 = 18
\]
**Distributive Law of Multiplication Using Variables:**

We can also use the distributive law when multiplying variables:

**Example:**  $3(4 + x)$

This expression has one monomial ($3$) and one binomial ($4 + x$). To simplify, we clear the parentheses. This means that we multiply each term in the binomial by the monomial, using the **distributive law of multiplication**:

$$3(4 + x) = 12 + 3x$$

**On Your Own:** Use the distributive law to simplify the expressions.

1. $3x(4x + 5)$

$$3x(4x + 5) = _____________________$$

2. $-2x(3x - 12)$

$$-2x(3x - 12) = _____________________$$

3. $5x(3x - 12)$

$$5x(3x - 12) = _____________________$$

4. $-3(8x - 9)$

$$-3(8x - 9) = _____________________$$
Combining Like Terms

Another way to simplify expressions is to combine like terms. Look at the next example:

**Example:** \(3(4 + x) + x(4 + 5x) + 3\)

### Steps:

- Clear the parentheses, using the **distributive law**:

  \[
  (3 \cdot 4) + (3 \cdot x) + (x \cdot 4) + (x \cdot 5x) + 3 = \\
  12 + 3x + 4x + 5x^2 + 3
  \]

- **Combine like terms.** (Note: This means **add**.)

  **Like terms** refer to those terms that are similar, such as whole numbers or variables with the same exponents.

  \[
  12 \quad \text{and} \quad 3 \quad \text{are like terms!} \\
  12 + 3x + 4x + 5x^2 + 3 = 15 + 7x + 5x^2
  \]

  \[
  3x \quad \text{and} \quad 4x \quad \text{are like terms!}
  \]

  **Note:** Since there is only one term that is raised to the second power \((5x^2)\), we cannot add it to anything.

- Write the answer in proper order. Begin with variables raised to the **highest power** and descend to those with lower powers. End with the constant (i.e. the term without a variable):

  \[
  15 + 7x + 5x^2 = \underline{______________}
  \]
Order Is Important!

Notice that, in the answer to the above example, the terms were listed in a certain order:

\[ 5x^2 + 7x + 15 \]

We begin with the variables raised to the highest power and descend to those with lower powers. The constant (i.e. the term without a variable) comes last.

On Your Own

Example: Simplify \(7(2x^2 + 3) + 3(4 + x) + x^2\)

• Use the distributive law:
  
  \[(7 \cdot \_\_) + (7 \cdot \_\_) + (3 \cdot \_\_) + (3 \cdot \_\_) + x^2 = \]

• Add like terms.

• Write the answer in proper order. (Begin with variables raised to the highest power and descend to those with lower powers. End with the constant.)
Practice: Simplify the following expressions.

1. $5(3x - 4) + 3(x - 5) + 4x^2$

2. $-3(4x^2 - 5) + 3(-2x^2 + 5x) + 8$

3. $2x(x + 3) + 3(4x^2 - 5x + 3)$

4. $5(4x^2 + 3x - 2) + 2(6x^2 - 8x - 10)$

5. $2(3x + 5) + 3(7x - 4) + 3x^2$

6. $10(x^2 + 5) + 9(3x^2 + 4x + 5) + 10$

7. $x(x + 5) + x^2(x -3) + 9$

8. $4x^2(5x + 6) + 2x(x^2 + 10x - 12)$

9. $5(6x^3 + 5x + 7) + 3x^2$

10. $-3x(-3x - 4) + 4x(7x - 9) + 4x^2$
iii. Multiplying Binomial Expressions (FOIL)

When multiplying two binomial (two-term) expressions, we multiply each term in the first expression by each term in the second expression. One way to do this is by using the FOIL method. FOIL stands for First, Outer, Inner, Last. That is the order in which we multiply the terms. To demonstrate this concept, let's begin with a non-algebra example:

Example: $(20 + 3)(40 + 5)$

Note: When you apply the FOIL method, you end up with a smiley face:

- **First**: Multiply the first term in each binomial expression
  
  \[(20 + 3)(40 + 5)\]
  
  \[20 \cdot 40 = 800\]

- **Outer**: Multiply the outer term in each binomial expression
  
  \[(20 + 3)(40 + 5)\]
  
  \[20 \cdot 5 = 100\]

- **Inner**: Multiply the inner term in each binomial expression
  
  \[(20 + 3)(40 + 5)\]
  
  \[3 \cdot 40 = 120\]

- **Last**: Multiply the last term in each binomial expression.
  
  \[(20 + 3)(40 + 5)\]
  
  \[3 \cdot 5 = 15\]

Continued on next page
1.2

• The final step is to **combine** (add) like terms. In this case, all of our terms are integers so it is easy:

\[
\begin{aligned}
20 \cdot 40 &= 800 \\
20 \cdot 5 &= 100 \\
3 \cdot 40 &= 120 \\
3 \cdot 5 &= 15 \\
\end{aligned}
\]

\[
\underbrace{800 + 100 + 120 + 15}_{\text{add}} \quad \text{Sum}
\]

Let's do an example that has both integers (constants) and variables:

**Example:** \((3 + 7x)(6 + 2x)\)

Apply the FOIL method:

- Multiply the **first** term in each expression: \(3 \cdot 6 = 18\)

\[(3 + 7x)(6 + 2x)\]

- Multiply the **outer** terms in each expression: \(3 \cdot 2x = 6x\)

- Multiply the **inner** terms in each expression: \(7x \cdot 6 = 42x\)

- Multiply the **last** terms in each expression: \(7x \cdot 2x = 14x^2\)

- Add and combine any **like terms**: \(14x^2 + 6x + 42x + 18\)

Answer: ________________
Let's do another example:

**Example:** \((x + 2)(x + 3) = ____\)

Use the FOIL method to solve the above problem.

___ + ___ + ___ + ___ = ______________________

**Note:** We can also multiply two binomials by using a grid. Look at the example below.

**Example:** \((x + 2)(x + 3)\)

When we multiply each term in each row by each term in each column, we get the following: \(2x + 6 + x^2 + 3x\)

Now combine like terms: ___________________

Finally, place the terms in proper order (descending powers):

________________
On Your Own

1. \((x + 5)(x + 5) = \) _____

2. \((x + 5)(x - 5) = \)

3. \((x - 5)(x - 5) = \)

4. \((3x + 5)(2x + 3) = \)

5. \(2(3x + 2)^2 = \)
Mixed Practice: Show all work!

1. Simplify: $3(15x^2 + 3x + 8)$

2. Simplify: $5(3x^2 + x + 7) + 2x(x + 3) + 8$

3. Simplify: $4x(3x + 6) + 3(2x + 9) + 4x^2 + 3x + 7$

4. Simplify: $(3x + 4)(3x - 4)$

5. Simplify: $(4x - 5)(4x - 5)$
6. \((4x - 3)(x + 4) = \) _____
   A. \(4x^2 + 8x - 12\)
   B. \(4x^2 + 13x - 12\)
   C. \(4x^2 + 16x - 12\)
   D. \(4x^2 + 13x + 12\)

7. \((x + 3)(5x - 2) = \) _____
   A. \(5x^2 + 13x - 6\)
   B. \(5x^2 + 15x - 6\)
   C. \(5x^2 + 17x - 6\)
   D. \(5x^2 + 15x + 1\)

8. \((3x - 2)^2 = \) _____
   A. \(9x^2 - 4\)
   B. \(9x^2 + 4\)
   C. \(9x^2 - 12x - 4\)
   D. \(9x^2 - 12x + 4\)

9. \(2a(4a^2 + 6) = \) _____
   A. \(8a^3 + 8a\)
   B. \(8a^3 + 12\)
   C. \(8a^3 + 6\)
   D. \(8a^3 + 12a\)

10. \(2m^2 (5m^2 - 6m + 2) = \) _____
    A. \(10m^4 - 6m + 2\)
    B. \(10m^2 - 12m^3 + 4m^2\)
    C. \(10m^4 - 12m^3 + 4m^2\)
    D. \(10m^4 - 12m^3 + 2\)
C. Evaluating Algebraic Expressions

On the CAHSEE, you may be also asked to substitute values (numbers) in an algebraic expression and evaluate (solve) the expression.

i. Evaluating Expressions in One Variable

Example: Evaluate the expression $4x + 9$ when $x = 3$.

a. Replace each variable (letter) with its assigned value (number). Always use parentheses.

$$4(3) + 9$$ Parentheses remind you that it’s $4 \cdot 3$, not 43.

b. Perform the operations in the expression by applying the order of operations. (Note: Multiplication comes before addition!)

$$4(3) + 9 = 12 + 9 = 21$$

On Your Own

1. Evaluate the expression $6y + 15$ when $y = 2$.

   $6(\text{___}) + 15 = \text{___} + \text{___} = \text{______}$

2. Evaluate the expression $8 - 2x$ when $x = 3$.

   $8 - 2(\text{___}) = 8 - \text{___} = \text{____}$

3. Evaluate the expression $-2n + 5n$ when $n = -3$

4. Evaluate the expression $4y + 12$ when $y = -9$
ii. Evaluating Expressions in Two Variables

**Example:** Evaluate $3x^3 + 2x^2 - 3y$ when $x = 2$ and $y = 3$.

a. Replace each variable (letter) in the expression with its value (number).

$$3x^3 + 2x^2 - 3y = 3(2)^3 + 2(2)^2 - 3(3)$$

b. Perform operations in order of operations:

$$3(8) + 2(4) - 3(3) \quad \text{Exponents First}$$

$$24 + 8 - 9 \quad \text{Multiplication Second}$$

$$23 \quad \text{Addition & Subtraction (Left to Right)}$$

**Example:** Evaluate $3x^2 - 4xy - 3y$ when $x = 2$ and $y = -2$.

a. Replace each variable in the expression with its value.

$$3x^2 - 4xy - 3y = 3(2)^2 - 4(2)(-2) - 3(-2)$$

b. Perform operations in order of operations:

$$3(4) - 4(2)(-2) - 3(-2) \quad \text{Exponents}$$

$$12 + 16 + 6 = \quad \text{Multiplication}$$

_______ \quad \text{Addition}
On Your Own

**Example:** If \( x = 3 \) and \( y = 5 \), then \( x(5 + y^2) = \) _____.

a. Replace each variable in the expression with its value:

\[
x(5 + y^2) = 3(5 + 5^2)
\]

b. Perform operations in order: Do what’s in the parentheses first and in order (exponents before addition)

\[
3(5 + 5^2) = 3(5 + ___) = 3(____) = __________
\]

**Example:** Evaluate the expression \( \frac{3x + 3}{6} + y \) when \( x = 5 \) and \( y = 3 \).

a. Replace each variable with its numeric value:

\[
\frac{3(____) + 3}{6} + ___
\]

b. Perform operations in order:
Practice

1. Evaluate $x^2 + 5x + 3y$ when $x = -3$ and $y = 4$
   
   ________________________________

2. Evaluate $3x^2 + 4x + 2xy$ when $x = 4$ and $y = 5$.
   
   ________________________________

3. Evaluate $4x + 5y^2$ when $x = 9$ and $y = -2$.
   
   ________________________________

4. If $a = 4$ and $b = 7$, then $ab + 4 + 3a = _____$
   
   ________________________________

5. Evaluate $5x^3 - y^2 - 3$ when $x = 2$ and $y = 6$
   
   ________________________________

6. If $a = 10$ and $b = -4$, then $a^2 + b^2 = _____$
   
   ________________________________
7. Evaluate $3x(x + y^2)$ for $x = 3$ and $y = 4$.

8. If $a = 5$ and $b = 3$, then $a(2b + 4) = ____$

9. If $x = 4$ and $y = 5$, then $(x + y)^2 - 11 = ____$

10. If $m = 3$ and $n = -2$, then $\frac{mn + 18}{4} = ____$

11. If $x = 3$ and $y = 3$, then $\frac{x + y}{x} + y = ____$
Unit Quiz: The following problems appeared on the CAHSEE.

1. Simplify: $3(2x + 5)^2$
   
   A. $6x + 20x + 15$
   B. $6x^2 + 10x + 15$
   C. $12x^2 + 20x + 25$
   D. $12x^2 + 60x + 75$

2. If $n = 2$ and $x = \frac{1}{2}$, then $n(4 - x) = _____$
   
   A. 1
   B. 3
   C. 7
   D. 10

3. If $h = 3$ and $k = 4$, then $\frac{hk + 4}{2} - 2 = ________$
   
   A. 6
   B. 7
   C. 8
   D. 10
Unit III: Simplifying Complex Monomial Expressions

On the CAHSEE, you will be asked to simplify complex monomial expressions. Monomials expressions contain one term!

Example: \((4x^2z^2)\)

Example: \(\frac{18x^3y}{24xy^5}\)

Notice that these expressions require that you multiply variables containing exponents. Let’s do a quick review of exponents before we tackle these kinds of problems.

A. Exponents Review

Exponents are a shorthand way of representing how many times a number is multiplied by itself. The number being multiplied is called the base, and the exponent tells how many times the base is multiplied by itself. The format for using exponents is: \((\text{base})^{\text{exponent}}\)

Example: \(4^3 = 4 \cdot 4 \cdot 4 = 64\)

On Your Own

1. \(3^3 = \_\_ \cdot \_\_ \cdot \_\_ = \_\_\_\_\)

2. \(2^4 = \_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_ = \_\_\_\_\_\)

3. \(2^5 = \_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\)
When multiplying exponential expressions, keep the base and add the exponents.

Example: \(2^5 \cdot 2^4 = 2^{5+4} = 2^9\)

On Your Own

1. \(5^2 \cdot 5^8 = \) 
2. \(4^1 \cdot 4^7 = \)
3. \(5^{-3} \cdot 5^3 = \)
4. \(7^3 \cdot 7^3 = \)

When dividing exponential expressions, keep the base and subtract the exponents.

Example: \(\frac{3^5}{3^3} = 3^{5-3} = 3^2 = 9\)

On Your Own

1. \(\frac{2^7}{2^3} = \)
2. \(\frac{4^3}{4^3} = \)
3. \(\frac{5^3}{5^1} = \)
When an exponent is negative, the expression represents a fraction: the numerator is always equal to 1, and the denominator consists of the base and the absolute value of the exponent. When we convert the expression to a fraction, the exponent is positive.

Example: $2^{-3}$ means $\frac{1}{2^3}$

On Your Own

1. $3^{-1} = \underline{\hspace{2cm}}$
2. $3^{-2} = \underline{\hspace{2cm}}$
3. $x^{-2} = \underline{\hspace{2cm}}$
4. $4^{-2} = \underline{\hspace{2cm}}$
5. $3^{-3} = \underline{\hspace{2cm}}$
6. $(-3)^{-3} = \underline{\hspace{2cm}}$
7. $5^{-3} = \underline{\hspace{2cm}}$
8. $5^{-1} = \underline{\hspace{2cm}}$
9. $-5^{-2} = \underline{\hspace{2cm}}$
10. $\frac{1}{x^{-1}} = \underline{\hspace{2cm}}$
To understand why negative exponents represent fractions, look at the chart below:

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Simplified Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^3$</td>
<td>27</td>
</tr>
<tr>
<td>$3^2$</td>
<td>9</td>
</tr>
<tr>
<td>$3^1$</td>
<td>3</td>
</tr>
<tr>
<td>$3^0$</td>
<td>1</td>
</tr>
<tr>
<td>$3^{-1}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$3^{-2}$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>$3^{-3}$</td>
<td>$\frac{1}{27}$</td>
</tr>
</tbody>
</table>

What happens each time the exponent decreases by one?

Explain why $3^0$ equals 1:

Explain why $3^{-1}$ equals $\frac{1}{3}$:
**Practice:** Complete the chart. The first one has been done for you.

<table>
<thead>
<tr>
<th>Negative Exponent</th>
<th>Positive Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7^{-3}$</td>
<td>$\frac{1}{7^3}$</td>
</tr>
<tr>
<td>$(-5)^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$x^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{x^2}{x^{-3}}$</td>
<td></td>
</tr>
<tr>
<td>$2x^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{x^{-1}}$</td>
<td></td>
</tr>
<tr>
<td>$(3x)^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2x^{-4}}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{4x^{-2}}$</td>
<td></td>
</tr>
</tbody>
</table>
Practice: Write each expression in non-fractional form.

Example: \( \frac{1}{(2x)^{-4}} = 16x^4 \)

1. \( \frac{1}{x^2} = \) ____

2. \( \frac{4}{y^{-2}} = \) ____

3. \( \frac{3}{y^3} = \) ____

4. \( \frac{1}{(-3y)^{-3}} = \) ____
B. Multiplying Monomials

To multiply two or more monomial expressions, we need to first multiply like terms:

**Example:** \((4x^2z^2)(3xz^3)\)

- Multiply the coefficients (the numbers before the variables).
  \[4 \cdot 3 = 12\]

- Multiply the variables: Add the exponents with a common base.
  \[x^2 \cdot x = x^{2+1} = x^3\]
  \[z^2 \cdot z^3 = z^{2+3} = z^5\]

- Multiply all terms: \(12 \cdot x^3 \cdot z^5 = \) __________

**On Your Own**

1. \((5a^5b^3c^1)(9a^2b^4c^4)\)

- Multiply the coefficients (the numbers before the variables).
  \[5 \cdot 9 = \] __________

- Multiply the variables:
  \[a^5 \cdot a^2 = \] __________
  \[b^3 \cdot b^4 = \] __________
  \[c^1 \cdot c^4 = \] __________

Now multiply all terms together: _______________
Practice

1. Which expression is equivalent to \((5x^2)(4x^5)\)?
   A. \(9x^7\)
   B. \(9x^{10}\)
   C. \(20x^7\)
   D. \(20x^{10}\)

2. \((6x^6y^3z^7)(6x^5y^4z^2)\) = ______________________

3. Which expression is equivalent to \((12x^8)(-4x^2)\)?
   A. \(-48x^{10}\)
   B. \(-48x^{16}\)
   C. \(8x^{10}\)
   D. \(8x^{16}\)

2. Which expression is equivalent to \((4x)^3\)?
   A. \(12x^5\)
   B. \(12x^6\)
   C. \(64x^5\)
   D. \(64x^3\)

3. Simplify the expression shown below:

\[
(7m^8n^5)(8m^3n^4)
\]

4. Simplify the following expression: \((3x^3z^2)(5xz^3)\)
C. Dividing Monomials

To divide monomials, we need to first **divide all like terms** (so that the **same terms aren't** in the numerator and the denominator).

**Example:** \( \frac{18x^3y}{24xy^5} \)

**Steps:**
- Divide the **coefficients**. (Reduce coefficients by dividing out common factors):

  \[
  \frac{18}{24} = \frac{3}{4} \quad \frac{3}{6} = \frac{3}{6} = \frac{1}{2}
  \]

  Now we have \( \frac{3x^3y}{4xy^5} \)

- Divide the variables: Subtract the exponents (with a common base) that you find in both the numerator and denominator:

  \[
  \frac{x^3}{x} = x^{3-1} = x^2
  \]

  \[
  \frac{y}{y^5} = y^{1-5} = y^{-4} = \frac{1}{y^4}
  \]

- Multiply all the terms that go in the numerator. Then multiply all the terms that go in the denominator. Your answer should be a simpler fraction than that in the original problem. (No variable appearing in the numerator should appear in the denominator and vise-versa.)
On Your Own

1. Which expression is equivalent to \( \frac{-2n^2}{n} \) ?
   
   A. \( \frac{-2}{n} \)
   
   B. -2n
   
   C. \( \frac{2n}{n} \)
   
   D. \( \frac{-2n}{n} \)

2. Which expression is equivalent to \( \frac{3x}{5x^3} \) ?
   
   A. \( \frac{3x^2}{5} \)
   
   B. \( \frac{3}{5x^2} \)
   
   C. \( \frac{3x}{5} \)
   
   D. \( \frac{3}{5x} \)

3. \( \frac{(3a^3)(4a^4)}{a^2} = \) ________________________
4. \( \frac{(2x^4y^5)^2}{x^2y^5} = \) ______________________________

5. \( \frac{15a^2b^5c^3}{40ab^4c^4} = \) ______________________________

6. \( \frac{16m^4n^5}{-4m^9n^{-3}} = \) ______________________________

7. Which expression is equivalent to \( 12x^{10} \div 3x \)?
   A. \( 9x^9 \)
   B. \( 9x^{10} \)
   C. \( 4x^9 \)
   D. \( 4x^{300} \)

8. Which expression is equivalent to \( 36x^8 \div 9x^7 \)?
   A. \( 4x \)
   B. \( x^{15} \)
   C. \( 4 \)
   D. \( 4x^{15} \)

9. Simplify and reduce: \( \frac{27a^3b^5c^2}{18a^4bc^5} = \) ______________________________
D. Multiplying and Dividing Fraction Monomials

When multiplying and dividing monomial expressions involving fractions, **multiply and divide like terms:**

### i. Multiplication

**Example:** \( \frac{9x^2y}{3xy} \cdot \frac{9x^3}{2y^2} \)

- Multiply the numerators together and multiply the denominators together:

\[
\frac{9x^2y}{3xy} \cdot \frac{9x^3}{2y^2} = \frac{9x^2y \cdot 9x^3}{3xy \cdot 2y^2} = \frac{81x^5y}{6xy^3}
\]

- Simplify fractions wherever possible.

  o Divide out common factors in the coefficients: \( \frac{81 \div 3}{6 + 3} = \underline{---------} \)

  o Divide variables. Subtract the exponents (with a common base) that you find in both the numerator and denominator:

\[
\frac{x^5}{x} = x^{5-1} = \underline{-----}
\]

\[
\frac{y}{y^3} = y^{1-3} = \underline{-----}
\]

- Place the simplified numerator over the reduced denominator:

\[
\underline{-----} \quad \underline{\text{reduced numerator}} \quad \underline{\text{reduced denominator}}
\]
On Your Own

1. \( \frac{4a^4b^2}{6ab^4} \cdot \frac{4a}{2a^2} = \) ______________

   - Multiply numerators together & multiply denominators together:

   - Simplify fractions wherever possible.
     a. Divide out common factors in the coefficients:

     - Place simplified numerator over reduced denominator:

   b. Divide variables. Subtract the exponents (with a common base) that you find in both the numerator and denominator:

  2. \( \frac{5m^3n^2}{2n^2} \cdot \frac{3mn^3}{4mn^2} = \) _________

  3. \( \frac{4a^5b}{3a^2b^2} \cdot \frac{3b^2c^3}{2b} = \) _________
ii. Division

Example: \( \frac{10ab}{4a^2} \div \frac{25b^3}{7ab^2} \)

**Reminder:** Whenever you divide by a fraction, **flip, or invert** the second fraction (the divisor) and **change the sign** (from division to multiplication):

1. Change the sign and invert the second fraction: \( \frac{10ab}{4a^2} \cdot \frac{7ab^2}{25b^3} \)

2. Follow the multiplication rules and solve:

**On Your Own**

1. \( \frac{6x^4y^5}{5xy^2} \div \frac{3x^3y}{4x^2y^2} = \) _____________
   
   • Invert the second fraction: ______________

   • Follow the multiplication rules and solve:

2. \( \frac{8x^5y^4}{6x^2y^3} \div \frac{6x^2y}{5xy^3} = \) ______

3. \( \frac{4ab^2c}{2ab} \div \frac{2ab}{ac^2} = \) ______
CAHSEE Distracters:

On the CAHSEE, there may be answer choices that are there to distract you. These choices are called "distracter items." Some of these distracters will involve fractions that cannot be reduced. Look at the following examples and place a check mark beside the algebraic fractions that can be simplified.

\[
\frac{x-5}{5} \quad \frac{4(x-2)}{16} \quad \frac{3x+6}{2y} \\
\frac{x(x-3)}{3x} \quad \frac{3x+4}{4} \quad \frac{x^2-5x}{x^2}
\]

What must the fraction contain so that it can be reduced?

_____________________________________________________

Now go back to the fractions that you checked above. Write one in each box and simplify it:
On Your Own

Look at the problem: Simplify \( \frac{2x - 4}{2x} \)

Cross out all of the distracter items below and circle the correct choice.

A. -4

B. \( \frac{x - 2}{x} \)

C. \( x - 2 \)

D. \( \frac{2}{x} \)

Look at the next problem: Simplify \( \frac{4x + 6}{4x} \)

Cross out all distracter items below and circle the correct choice:

A. 6

B. \( x + 6 \)

C. \( 2x + 3 \)

D. \( \frac{2x + 3}{2} \)
E. Taking the Roots of Monomials

On the CAHSEE, you will be asked to find the root of an algebraic monomial expression:

Example: \( \sqrt{16x^2} \)

Before we proceed with this type of problem, let’s do a quick review of square roots and cube roots.

Square Roots

The square root of a number is one of its two equal factors:

Example: \( 8^2 = 8 \cdot 8 = 64 \), so \( \sqrt{64} = ____ \)

Note: The square of any whole number is called a perfect square.

List the first ten perfect squares:

\[
1 \quad 4 \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \\
1 \cdot 1 \quad 2 \cdot 2
\]

Cube Roots

The cube root of a number is one of its three equal factors:

Example: \( 5^3 = 5 \cdot 5 \cdot 5 = 125 \), so \( \sqrt[3]{125} = ____ \)

Note: The cube of a whole number is called a perfect cube.

The first five perfect cube roots are \( 1 \quad 8 \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \quad ____ \\
1 \cdot 1 \cdot 1 \quad 2 \cdot 2 \cdot 2 \)
Finding the Root

Roots are represented by the symbol $\sqrt{}$. This symbol is called the radical.

**Example: $\sqrt{81}$**

This means "the square root of 81."

To find the square root of 81, ask, "What number when multiplied by itself is equal to 81?" Since $9 \cdot 9 = 81$, then 9 is the **square root** of 81.

**Example: $\sqrt[3]{27}$**

This means "the cube root of 27."

To find the cube root of 27, ask, "What number when multiplied by itself three times is equal to 27?" Since $3 \cdot 3 \cdot 3 = 27$, then 3 is the **cube root**.

**We can find the roots of algebraic expressions in the same way.** Break the expression (radicand) into known **perfect powers**.

**Example: $\sqrt{x^2} = \sqrt{x \cdot x} = x$**

Now, to find the root of an expression that contains both a coefficient and a variable, we take the root of each separately:

**Example: $\sqrt{100x^2} = \sqrt{10^2 \cdot x^2} = \sqrt{10^2} \cdot \sqrt{x^2} = 10x$**

Now, let's go back to the original problem: $\sqrt{16x^2}$

$\sqrt{16x^2} =$
On Your Own

1. $\sqrt{49x^4} = _____$
2. $\sqrt{144y^2} = _____$
3. $\sqrt{121m^4} = _____$
4. $\sqrt{169x^2} = _____$
5. $\sqrt{36a^4b^{16}} = _____$
6. $\sqrt{25x^2y^2z^2} = _____$

Unit Review

1. $5 \cdot 5^4 = _____$
2. $5^0 \cdot 5^4 = _____$
3. Simplify the following expression: $\frac{5^3}{5^8} = _____$
4. Simplify the following expression: $\frac{5^{-3}}{5^{-8}} = _____$
5. Simplify the following expression: $\frac{5^3}{5^{-8}} = _____$
6. Simplify the following expression: $3 \cdot \sqrt{64} = \ldots$

7. $\sqrt[3]{64} = \ldots$

8. $\frac{3ab}{5ab^2} \div \frac{4a^3b}{5ab^4} = \ldots$

9. $(8y^4)(-7y^{-2}) = \ldots$
Unit Quiz: The following problems appeared on the CAHSEE.

1. Simplify the expression: $2x^{-3}$
   
   A. $\frac{8}{x^3}$
   
   B. $(2x)^{-1}(2x)^{-1}(2x)^{-1}$
   
   C. $\frac{2}{x^3}$
   
   D. $\frac{1}{(2x)^3}$

2. $x^3y^3 =$
   
   A. $9xy$
   
   B. $(xy)^6$
   
   C. $3xy$
   
   D. $xxxxyy$

3. Simplify the expression shown below.
   
   $(5x^2z^2)(8xz^3)$
   
   A. $40x^2z^6$
   
   B. $40x^3z^5$
   
   C. $40x^3z^6$
   
   D. $40x^5z^5$
4. Simplify the expression shown below.

\[(6a^4bc)(7ab^3c)\]

A. \(13a^4b^3c\)
B. \(13a^5b^4c^2\)
C. \(42a^4b^3c\)
D. \(42a^5b^4c^2\)

5. Find \(\sqrt{4x^4}\)

A. 2
B. 2x
C. 4x
D. 2x²
Unit IV: Foundational Concepts for Algebraic Equations

On the CAHSEE, you will be asked to solve linear equations and inequalities in one variable.

The key to solving any equation or inequality is to get the unknown variable all by itself. When isolating a variable, it is important to remember that equations must balance at all times. Therefore, whatever you do to one side of an equation to isolate the unknown variable, you must do to the other side as well.

Example: If \(3x = 12\), then \(x\) must be equal to 4.

Note: To get the unknown variable, \(x\), all by itself, we divide both sides by 3.

There are three important concepts that come into play when solving algebra equations: the sum of the opposites, the product of reciprocals, and the distributive property. In this unit, we will look at each of these concepts and examine how they are applied in algebra.

A. Opposites

Any set of integers that are the same distance from zero but in opposite directions (and have opposite signs) are called opposites. A number's opposite is its additive inverse.

Rule: To take the opposite of the number, simply change the sign.

Example: The integers \(-3\) and \(+3\) are opposites.
On Your Own: Find the opposite of each number.

<table>
<thead>
<tr>
<th>Number</th>
<th>Its Opposite</th>
</tr>
</thead>
<tbody>
<tr>
<td>195</td>
<td>-195</td>
</tr>
<tr>
<td>-195</td>
<td>195</td>
</tr>
<tr>
<td>-47</td>
<td>47</td>
</tr>
<tr>
<td>47</td>
<td>-47</td>
</tr>
<tr>
<td>-1,359</td>
<td>1,359</td>
</tr>
<tr>
<td>1,359</td>
<td>-1,359</td>
</tr>
</tbody>
</table>

Adding Opposites

**Rule:** When we add any number and its opposite, we get 0.

**Example:** 
-15 + (15) = 0

**Example:** 
35 + (-35) = 0

On Your Own

1. -45 + ____ = 0

2. If y = 23, then -y = ______

3. If 334 + x = 0, then x = _____

4. ____ - 635 = 0

5. _____ + 34 = 0
Why Is the Concept of Opposites Important in Algebra?

When solving an algebra problem, the goal is to find the value of the unknown variable. We do that by getting that variable all by itself. We must remove everything around it. One way we do this is to take the opposite of a number that is added to or subtracted from that variable, and to add this opposite to both sides.

Remember: When we add a number and its opposite, we get 0.

Look at the next example:

\[ x + 15 = 11 \]

\[ +15 + (-15) = 0 \]

\[ x + 15 + (-15) = 11 + (-15) \quad \text{Add the opposite to get } x \text{ alone!} \]

\[ x + 15 - 15 = 11 - 15 \quad \text{Note: Adding } -15 \text{ means subtracting } 15. \]

\[ x = -4 \]

**On Your Own**

\[ x - 13 = 20 \]

\[ \underline{\quad } \quad \text{Add the opposite to get } x \text{ alone!} \]

\[ x = \underline{\quad } \]

\[ x + 13 = 20 \]

\[ \underline{\quad } \quad \text{Add the opposite to get } x \text{ alone!} \]

\[ x = \underline{\quad } \]
Practice: Solve for $x$.

1. $x + 14 = 20$

2. $-5 + x = 15$

3. $x - 13 = 11$

4. $x - 8 = -8$

5. $-9 + x = -15$
B. Reciprocals

The reciprocal of a number is its multiplicative inverse. The numerator and denominator are inverted. This property is often used in equation solving when you want a number to cancel out.

**Rule:** When we multiply a number by its reciprocal, we get a product of 1.

To find the reciprocal of a fraction, invert the numerator and denominator.

**Example:** The reciprocal of \( \frac{4}{5} \) is \( \frac{5}{4} \)

**Example:** The reciprocal of \( \frac{1}{2} \) is \( \frac{2}{1} \), or 2.

**Note:** Zero does not have a reciprocal.

Finding the Reciprocal of a Whole Number

To find the reciprocal of a whole number, place it over 1 (to make a fraction), and then invert the fraction.

**Example:** Find the reciprocal of 6.

- Place it over 1: \( \frac{6}{1} \)
- Now flip it: \( \frac{1}{6} \)
On Your Own: Find the reciprocal of each number:

\[
\begin{array}{cccc}
\frac{3}{8} & -\frac{3}{8} & -28 \\
28 & -650 & -\frac{1}{8}
\end{array}
\]

Multiplying Reciprocals

Rule: Whenever we multiply two numbers that are reciprocals of one another, the product equals 1.

Example: 8 and \(\frac{1}{8}\) are reciprocals because \(8 \cdot \frac{1}{8} = 1\).

Example: \(\frac{4}{9}\) and \(\frac{9}{4}\) are reciprocals because \(\frac{4}{9} \cdot \frac{9}{4} = 1\).

On Your Own

\[
\begin{array}{cccc}
\frac{1}{5} \cdot 5 & -\frac{3}{4} \cdot -\frac{4}{3} & -36 \cdot ___ & 1 \\
-\frac{2}{5} & 270 \cdot ___ & 3 \cdot ___ & 1
\end{array}
\]
Why Are Reciprocals Important in Algebra?

The concept of finding reciprocals is essential in algebra. As we mentioned earlier, the goal in solving an algebra equation is to find the value of the unknown variable. We do this by removing everything around it. We learned that adding opposites is one way to do this. Another way is to separate the variable from its coefficient (the numeric factor of the term).

Example: The term $5x$ is made up of the coefficient (5) and the unknown variable ($x$).

Since the product of any number and its reciprocal is 1, we can isolate $x$ by multiplying the coefficient by its multiplicative inverse (or reciprocal):

$$5 \cdot \frac{1}{5} = 1$$

Example: $5x \cdot \frac{1}{5} = 1x = x$

Now look at the following equation:

Example: $5x = 45$

Remember: The equation must remain balanced at all times. Therefore, if we multiply one side of the equation by a certain value, we must multiply the other side of the equation by the same value.

$$5x \cdot \frac{1}{5} = 45 \cdot \frac{1}{5} \quad \text{Multiply both sides by } \frac{1}{5}$$

$$1x = \frac{45}{5} \quad \text{Multiplying by } \frac{1}{5} \text{ is the same as dividing by 5.}$$

$x = \underline{9}$
Look at the next example:

**Example:** \( \frac{x}{5} = 7 \)

Here, the **coefficient** is \( \frac{1}{5} \). (Note: \( \frac{1}{5} x = \frac{1x}{5} = \frac{x}{5} \).)

The **reciprocal** of \( \frac{1}{5} \) is 5. To isolate \( x \) and keep the equation in balance, **multiply** both sides by 5:

\[
\frac{x}{5} = 7 \\
5 \left( \frac{x}{5} \right) = 5(7) \quad \text{Multiply both sides by 5.} \\
x = ____
\]

Look at the next example:

**Example:** \( -\frac{x}{5} = 11 \)

Here, the **coefficient** is \( -\frac{1}{5} \) and its **reciprocal** is -5. To isolate \( x \) and keep the equation in balance, we must **multiply** both sides by -5:

\[
-\frac{x}{5} = 11 \\
-5\left( -\frac{x}{5} \right) = -5(11) \quad \text{Multiply both sides by -5} \\
x = ____
\]
On Your Own: Solve for x.

1. \(6x = 42\)

2. \(-6x = -48\)

3. \(\frac{x}{5} = -12\)

4. \(-\frac{x}{3} = 9\)

5. \(9x = 54\)
C. Distributive Property

According to the distributive law, multiplication is distributed to each term in parentheses.

**Example:** \( 4 \left( 3 + 7 \right) = (4 \cdot 3) + (4 \cdot 7) = 12 + 28 = ____ \)

In the above example, the number outside of the parentheses (4) is multiplied by each term within the parentheses: 3 and 7.

**Note:** We get the same result by following the order of operations:

\[ 4(3 + 7) = 4 \cdot ____ = ____ \]

**On Your Own:** Simplify each expression, using both the distributive law and the order of operations.

1. Simplify: \( 4(3 - 7) = (4 \cdot __) - (4 \cdot __) = ________ \)
   
   Now solve using the order of operations:
   
   \[ 4 \left( 3 - 7 \right) = 4 \cdot (____) = ____ \]

2. Simplify -5(3 + 7) = _________________________________
   
   Now use the order of operations: _______________________

3. Simplify: 7(8 - 6) = _________________________________
   
   Now use the order of operations: _______________________

4. Simplify: -5(9 + 4) = _________________________________
   
   Now use the order of operations: _______________________
Using the Distributive Law in Algebra

The distributive law is also used with variables:

**Example:** 6(3x + 4) = (6 \cdot 3x) + (6 \cdot 4) = 18x + 24 = 18x + 24

**Practice:** Use the distributive law to simplify each of the expressions:

1. \[ 7(4x + 3) = \phantom{18x + 24} \]

2. \[ -4(-8x + 9) = \phantom{18x + 24} \]

3. \[ 9(-x - 7) = \phantom{18x + 24} \]

4. \[ -11(2x + 4) = \phantom{18x + 24} \]

5. \[ -8(9x + 8) = \phantom{18x + 24} \]

6. \[ -1(-x - 38) = \phantom{18x + 24} \]
Unit Quiz

1. Solve for $x$: $x - 14 = 27$

2. Solve for $x$: $4x = 28$

3. Solve for $x$: $-4x = 48$

4. Solve for $x$: $-5 + x = 16$

5. Solve for $x$: $-5 - x = 16$

6. Solve for $x$: $-3x = -15$

7. Simplify the expression $-3(3x - 9)$

8. Simplify the expression $2(-3x - 3)$

9. Simplify the expression $-4(-6x + 9)$

10. Simplify the expression $-5(-8x - 9)$
Unit V: Solving Linear Equations & Inequalities

Now that we are familiar with the concept of opposites and reciprocals and can apply the distributive law, we are ready to solve algebraic equations.

A. Equations

As we have learned, solving equations is just a matter of undoing operations and isolating the variables. Whatever we do to one side, we must do to the other. The goal is to get the variable all by itself!

Example: Solve for $x$: $x - 5 = 8$

To isolate the variable $x$, apply the rule of opposites (add 5 to both sides of the equation):

$x - 5 + 5 = 8 + 5 \quad \text{Apply the rule of opposites!}$

$x = 13$

On Your Own: Solve for $x$ by applying the rule of opposites.

1. $x + 8 = 40$

   $x + 8 + \underline{\text{___}} = 40 + \underline{\text{___}} \quad x = \underline{\text{____}}$

2. $x - 3 = 15 \quad x = \underline{\text{____}}$

3. $3 + x = 18 \quad x = \underline{\text{____}}$

4. $14 + x = 15 \quad x = \underline{\text{____}}$
In the next problem, \(x\) is on both sides of the equation:

**Example:** Solve for \(x\):

\[
4x + 5 = x - 4
\]

**Steps:**

1. We need to move both terms with variable \(x\) to the same side of the equation. Apply the rule of opposites.

\[
\begin{align*}
4x + 5 &= x - 4 \\
-x &= -x \\
3x + 5 &= 0 - 4
\end{align*}
\]

\[\text{Subtract 1} \times \text{from both sides.}\]

2. We need to get both constants (5 and -4) on the same side of the equation. Apply the rule of opposites.

\[
\begin{align*}
3x + 5 &= -4 \\
-5 &= -5 \\
3x &= -9
\end{align*}
\]

**Note:** We want to get all \(x\)'s on one side and all constants on the other side.

3. We now have the equation \(3x = -9\). To find out what \(1x\) is equal to, apply the rule of reciprocals.

\[
\frac{3x}{3} = \frac{-9}{3} \quad \text{Multiply by the reciprocal of 3: } \frac{1}{3}
\]

\[x = ___\]

**Note:** When isolating the variable, we begin with addition or subtraction (adding opposites to get a sum of 0) and then move on to multiplication or division (multiplying reciprocals to get a product of 1). We can think of this as doing the order of operations backwards.
On Your Own: Solve for $x$.

1. $3x - 5 = 2x + 8$

\[
\begin{align*}
3x &- 5 = 2x + 8 \\
3x &+ 5 = ______ &Apply the rule of opposites!
\end{align*}
\]

$3x + 0 = ______$

\[\text{Apply the rule of opposites!}\]

Now how will you get $x$ all by itself?

2. $6 - x = -5x + 4$

\[
\begin{align*}
6 &- x = -5x + 4
\end{align*}
\]

3. $8 + x = 14 - x$

\[
\begin{align*}
8 &+ x = 14 - x
\end{align*}
\]
4. $14x - 13 = 11 + 8x$

5. $16 - x = 8 - 5x$

6. $3 + 3x = 12 - 15x$
7. $3x - 4 = 8$

8. $2x - 5 = 17$

9. $\frac{x}{3} + 2 = 5$
B. Inequalities

We examined inequalities in Unit 1. An inequality is a mathematical statement in which one number or variable is "not equal" to another.

**Example:** $7 < 9$
7 is not equal to 9. It is less than 9.

**Example:** $9 > 7$
9 is not equal to 7. It is greater than 7.

Let's do a quick review of the symbols for inequality:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>is less than</td>
<td>$&lt;$</td>
</tr>
<tr>
<td>is greater than</td>
<td>$&gt;$</td>
</tr>
<tr>
<td>is less than or equal to</td>
<td>$\leq$</td>
</tr>
<tr>
<td>is no more than</td>
<td>$\leq$</td>
</tr>
<tr>
<td>is greater than or equal to</td>
<td>$\geq$</td>
</tr>
<tr>
<td>is at least</td>
<td>$\geq$</td>
</tr>
</tbody>
</table>

Solving linear inequalities is pretty much the same as solving linear equations. We isolate the unknown variable.

**Example:** $x + 3 \geq 5$
\[
-3 \quad -3 \quad \text{Apply the rule of opposites!}
\]
\[
x \geq 2
\]

Here, we isolate $x$ by subtracting 3 from both sides.
Let's look at another example:

**Example:** \(4 + 2x < 8\)

\[
4 - 4 + 2x < 8 - 4 \quad \text{Subtract 4 from both sides.}
\]

\[
2x < 4
\]

\[
\frac{2x}{2} < \frac{4}{2} \quad \text{Divide both sides by 2.}
\]

\[
x < 2
\]

**On Your Own**

\[
2x - 6 < 2
\]

\[
6x + 3 > 15
\]
Exception to the Rule

There is one very important exception when solving linear inequalities:

When you multiply or divide an inequality by a negative number, it changes the direction of the inequality.

Example: Examine the following true statement: 5 > 3

What happens if we multiply both sides by -1? Fill in the blank with the correct inequality sign: (-1)(5) [square root] (3)(-1)

We see that the relationship between the two numbers has now changed and, therefore, we must change the inequality sign.

We can see this more clearly on a number line:

Example: Solve for x: 5 - 3x ≤ 13 + x

Steps:
- Subtract 5 from both sides: 5 - 5 - 3x ≤ 13 - 5 + x
  
  \(-3x ≤ 8 + x\)

- Subtract x from both sides: -3x - x ≤ 8 + x - x
  
  \(-4x ≤ 8\)

- Divide both sides by -4:
  
  \(-\frac{4x}{-4} ≤ \frac{8}{-4}\)

  \(x ≥ ___\) (Note: We change the sign.)

- Solution: x can be either ___ or any value greater than ___.

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On Your Own

1. \( 3x - 5 \leq 15 + 8x \)

2. \( 8x + 4 \leq 20 + 16x \)

3. \( -9x + 13 \geq -6x - 8 \)

4. Cecilia is saving for a new television. She saved $75 last month. She plans to add $50 each month until she has saved at least $400. Which inequality can be used to find \( m \), the number of months it will take her to save for her television?

   A. \( 50m - 75 \geq 400 \)
   B. \( 75 + 50m \geq 400 \)
   C. \( 50m - 75 < 400 \)
   D. \( 75m + 50 \geq 400 \)
   E. \( 75 + 50m < 400 \)
Unit Quiz: The following questions appeared on the CAHSEE.

1. Solve for \( x \): \(-3x + 9 > 30\)
   A. \( x > 21 \)
   B. \( x < -7 \)
   C. \( x > 21 \)
   D. \( x > 7 \)

2. Solve for \( n \): \(2n + 3 < 17\)
   A. \( n < 2 \)
   B. \( n < 3 \)
   C. \( n < 5 \)
   D. \( n < 7 \)

3. In the inequality \(2x + $10,000 \geq $70,000\), \( x \) represents the salary of an employee in a school district. Which phrase most accurately describes the employee’s salary?
   A. at least $30,000
   B. at most $30,000
   C. less than $30,000
   D. more than $30,000
4. Solve for $x$: $2x - 3 = 7$
   A. -5
   B. -2
   C. 2
   D. 5

5. The table below shows values for $x$ and corresponding values for $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

Which of the following represents the relationship between $x$ and $y$?

A. $y = \frac{1}{7}x$
B. $y = 7x$
C. $y = x - 6$
D. $y = x - 18$
Unit VI: Word Problems

On the CAHSEE, you will be asked to translate word problems into algebraic equations and solve. These problems may involve rate, distance, time and average speed. We will look at each of these problems in this unit.

A. Rate

A rate is a ratio of related qualities that have different units.

Speed is a rate involving distance and time, such as miles per hour.

Example: Jacques drove at a rate of 60 miles per hour.

Work productivity can be expressed as a rate.

Example: The secretary types at a rate of 60 words per minute.

Price can be expressed as a rate:

Example: Apples cost $1.56 per pound.

Notice that the word “per” appeared in each of the examples above. “Per” means “for each” or “for every” and implies division:

Example: Price per pound can be written as \( \frac{\text{price}}{\text{pound}} \)

What’s important in solving a rate problem is to be sure that you place each piece of information in the right slot of the right formula.
i. Distance

Distance can be measured by multiplying how fast you’ve been moving (rate) by how long you’ve been moving (time).

**Distance Formula:** Distance = Rate \( \cdot \) Time

\[ D = r \cdot t \]

**Think DIRT:** Distance is Rate \( \cdot \) Time

\( D = \text{Distance (Distance traveled from start to finish)} \)

\( r = \text{Rate (Speed or Pace)} \)

\( t = \text{Time (How long the person/object has been in motion)} \)

**Example:** If John drives 50 miles per hour, how many miles can he drive in 3 hours?

Distance = Rate \( \cdot \) Time

Distance = 50 \( \cdot \) 3 = 150 miles

**Example:** If John drives 50 miles per hour, how long will it take him to travel 300 miles?

Distance = Rate \( \cdot \) Time

300 miles = 50 \( \cdot \) Time

Represent the unknown variable, time, with the letter \( t \).

\[ 300 = 50t \]

Set up equation.

\[ \text{Divide both sides by } ____ \]

\[ t = ____ \]

It will take John ____ hours to drive 300 miles.
Example: If John drives 560 miles in 8 hours, how fast does he drive per hour?

This problem asks us to find how fast John drives. This means that we need to find the rate (miles per hour).

There are two ways to solve this problem:

**Method 1: Simplify**

His speed is given as a ratio of 560 miles per 8 hours. To find the rate in miles per hour (how many miles he travels in 1 hour), we need to express this ratio in lowest terms.

\[
\frac{560}{8} = \boxed{\quad} \quad \text{The rate is \quad miles per hour.}
\]

**Note:** To simplify a ratio, treat it as a fraction.

**Method 2: Use the Distance Formula**

We are given two of the three variables for the distance formula:

Distance = \quad

Rate = r \quad \text{This is what we are solving for!}

Time = \quad

We can use the distance formula:

Distance = \text{Rate} \cdot \text{Time}

\[
\quad = \frac{\quad}{\quad} \quad \text{Plug in known values & unknown variables}
\]

\[
\quad = \quad \cdot \quad \quad \quad \text{Solve for } r!
\]

John drives at a rate of ____ miles per hour.
Two-Step Problems

Example: If Doreen can drive 600 miles in 12 hours, how long will it take her to drive 900 miles?

This problem involves two steps:

Step 1: Find the rate (in miles per hour).

\[
\frac{60}{12} = ___
\]

Rate is ___ miles per hour.

Step 2: Apply the distance formula and solve for \( t \).

Distance = Rate \( \times \) Time

___ = __ \( t \)

\[
t = ___ \quad \text{Doreen can drive 900 miles in ___ hours.}
\]
On Your Own

1. An airplane flies 678 miles from Seattle to San Francisco. The trip takes an hour and a half. What is the airplane’s speed in miles per hour?

\[ D = \underline{\phantom{000}} \]
\[ r = \underline{\phantom{000}} \]
\[ t = \underline{\phantom{000}} \]

\[ \underline{\phantom{000}} = \underline{\phantom{000}} \cdot \underline{\phantom{000}} \]

Solve for \( r \).

The airplane's speed is _____ miles per hour.

2. Rosalind drove 128 miles in 2 hours. If she continued traveling at the same speed, how long would it take her to drive 384 miles?
3. Mark drove 7 hours at the rate of 65 miles per hour. How far did he travel?

4. Thomas drove 504 miles. If his average speed was 72 miles per hour, how long did the trip take in total?

5. An airplane travels at a constant speed of 400 miles per hour. How long will it take to reach his destination, which is 2400 miles away?
ii. **Worker Productivity**

Rate problems can also involve **worker productivity**.

**Example:** If Leila can sew four blankets in 6 days, how long will it take her to sew 12 blankets?

This problem involves the **rate of work**. There are **two** ways to solve this problem:

- With Algebra
- Without Algebra

**With Algebra**

Set up a **proportion** using two **ratios**.

A **proportion** is an **equation** that states that two **ratios** are **equal**.

The relationship between blankets and days in the first ratio (blankets to days) is equal to the relationship between blankets and days in the second ratio!

<table>
<thead>
<tr>
<th>blankets</th>
<th>new blankets</th>
<th>←← ←← Numerator is blankets in both ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>days</td>
<td>new days</td>
<td>←← ←← Denominator is days in both ratios</td>
</tr>
</tbody>
</table>

One of the quantities in the second ratio is the **unknown variable** and can be represented by the letter **x**. To solve, we **cross multiply** and then **solve for x**.

Before solving this problem, let's do a quick review of cross multiplication and then examine how it is used in proportions.
Cross Multiplication

The term "cross multiplication" is really a shorthand way of expressing the mathematical law of proportions, which states that, "In any proportion, the **product of the means** (the numerator of the second ratio multiplied by the denominator of the first ratio) is **equal to** the **product of the extremes** (the numerator of the first ratio multiplied by the denominator of the second ratio)."

Example:

\[
\frac{3}{5} = \frac{6}{10} \\
3 \cdot 10 = 5 \cdot 6
\]

Example: \( \frac{1}{2} = \frac{5}{10} \)

**Cross multiply:**

\[
\text{Numerator}_1 \cdot \text{Denominator}_2 = \text{Numerator}_2 \cdot \text{Denominator}_1:
\]

\[
1 \cdot 10 = 5 \cdot 2
\]

We know that the proportion \( \frac{1}{2} = \frac{5}{10} \) is true because when we cross multiply, we get two equal values!

We can use cross multiplication to check whether a proportion is true:

**Example:** \( \frac{2}{3} = \frac{10}{15} \)

\( 2 \cdot 15 = 3 \cdot 10 \) ← Yes, both products are equal to 30!
Cross Multiplication & Algebra

We can also use cross multiplication to find an unknown variable in an algebra problem that contains a proportion.

**Example:** Solve for $x$.

\[
\frac{x}{15} = \frac{3}{5}
\]

\[
\begin{array}{c}
\frac{x}{15} = \frac{3}{5} \\
\text{means} \\
\frac{x}{15} = \frac{3}{5} \\
\text{extremes}
\end{array}
\]

\[
5x = 45 \quad \text{Now divide to solve for } x.
\]

\[
x = 9
\]

Let’s look at two more examples:

\[
\frac{10}{15} = \frac{x}{45}
\]

\[
15x = 450 \quad \text{We cross multiply!}
\]

\[
\begin{array}{c}
\frac{15x}{15} = \frac{450}{15} \\
\text{We divide both sides by 15 to isolate the } x.
\end{array}
\]

\[
x = ___
\]

**Example:** \[
\frac{1}{2} = \frac{x}{50}
\]

\[
1 \cdot 50 = 2 \cdot x
\]

\[
50 = 2x
\]

\[
\frac{50}{2} = x
\]

Therefore, \[
\frac{1}{2} = \frac{50}{2}
\]
On Your Own: Use the method of cross multiplication to solve for $x$.

1. \( \frac{4}{3} = \frac{20}{x} \)

2. \( \frac{8}{x} = \frac{4}{5} \)

3. \( \frac{x}{3} = \frac{15}{9} \)

4. \( \frac{15}{7} = \frac{90}{x} \)
Now we are ready to tackle the problem on page 90:

If Leila can sew four blankets in 6 days, how long will it take her to sew 12 blankets?

Steps:

1. Establish the two ratios:

   1\textsuperscript{st} Ratio: \[
   \frac{4 \text{ blankets}}{6 \text{ days}}
   \]

   2\textsuperscript{nd} Ratio: \[
   \frac{12 \text{ blankets}}{x}
   \] \hspace{0.5cm} \text{This is our unknown variable (# days)}

2. Set up a proportion in which both ratios are equal:

   \[
   \text{Proportion: } \frac{4}{6} = \frac{12}{x}
   \]

3. Cross multiply:

   \[
   \frac{4}{6} = \frac{12}{x}
   \]

   \[4x = 72 \hspace{0.5cm} \text{Cross multiply!}\]

4. Isolate \(x\) by dividing both sides of the equation by 4:

   \[
   \frac{1}{4}x = \frac{72}{4}
   \]

   \[x = ___ \hspace{0.5cm} \text{Leila can sew 12 blankets in ___ days.}\]
Without Algebra

Let’s take a look at this problem again:

If Leila can sew four blankets in 6 days, how long will it take her to sew 12 blankets?

To solve without algebra, simply find her rate for 1 blanket and then multiply this rate by 12:

**Step 1: Find the rate**

| Rate: 6 days for 4 blankets: $\frac{\text{6}}{\text{4}} = \frac{\text{3}}{\text{2}} = 1.5$ |
| It takes $1\frac{1}{2}$ days per blanket |

**Step 2: Multiply the rate by 12:**

$1.5 \cdot 12 = ___$  or  $\frac{\text{3}}{\text{2}} \cdot 12 = ___$

We get the same answer that we got on page 94!

Let’s look at another example:

**Example:** A worker can dig a 40-foot well in 4 days. If he continues to work at the same pace, how long would it take him to dig a 60-foot well?

<table>
<thead>
<tr>
<th>With Algebra</th>
<th>Without Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\text{40}}{\text{4}} = x$</td>
<td>40-foot well in 4 days = ___-foot well in 1 day (Rate)</td>
</tr>
<tr>
<td>$40x = ___$</td>
<td>60-foot well $\rightarrow 10 \cdot ___$</td>
</tr>
<tr>
<td>$x = ___$</td>
<td>Worker needs ___ days.</td>
</tr>
</tbody>
</table>

Which method do you prefer? __________________________
iii. Cost

Rate problems can also involve **cost**.

**Example:** If 4 pens cost $4.16, how much do 11 pens cost?

<table>
<thead>
<tr>
<th>With Algebra</th>
<th>Without Algebra</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
4 \cdot x &= 11 \\
4.16 &= x \\
Cross multiply: & \\
4 \cdot (x) &= 4.16 \cdot (11)
\end{align*}
\]
| Cost of 1 pen: $4.16 ÷ 4 = ____ |
| Cost of 11 pens: ____ X 11 = $ ____ |
| 4x = ____                      | 11 pens cost $____             |
| x = ____                      |                                |

iv. Earnings

Rate problems can also involve **earnings**.

**Example:** Melanie earns $150 every 30 days. If she saves all of her money, how long will it take her to earn $600?

<table>
<thead>
<tr>
<th>With Algebra</th>
<th>Without Algebra</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
150 \cdot x &= 600 \\
30 &= x \\
150x &= ____ \\
x &= ____
\end{align*}
\]
| Dollars earned per day: $150 ÷ 30 = ____ |
| Days needed to save $600: 600 ÷ ____ = ____ |
| Melanie can earn $600 in ____ days. |
On Your Own: Solve the following rate problems, either with or without algebra.

1. John works 30 hours a week and is paid $195.00. How much does he earn per hour?

2. If six cans of tuna cost $7.20, how much do 14 cans cost?

3. It takes George 14.4 hours to reprogram 4 computers. If he continues to work at the same rate, how long will it take him to reprogram 15 computers?

4. Heidi earns $56 for every 4 hours at her waitress job at Luke’s Pizza. If she works 5 days per week, 8 hours each day, how much will she earn in two weeks?
B. Average Speed

Speed is another way of expressing rate, and average speed problems are a variation of distance problems. One difference in average speed problems is that we are now looking for the speed (or rate) rather than the distance. To get the formula for rate, divide both sides of the distance formula by \( t \):

\[
\text{Distance} = \text{Rate} \times \text{Time}
\]

\[
d = rt
\]

\[
\frac{d}{t} = \frac{rt}{t}
\]

\[
R = \frac{d}{t} \quad \text{This is the formula we will use!}
\]

There is another important difference in average speed problems. We are generally given the speed for two separate intervals of a trip and asked to find the average. To do this, we calculate the total distance covered and then divide this by the total time traveled.

\[
\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}
\]

**Example:** Darius travels from Sacramento to San Francisco. For the first hour, he drives at a constant speed of 50 miles per hour. Then he increases his speed to 60 miles per hour for the next hour. What was his average speed?

The trick here is to think of this as a “statistics” problem and to apply what you know about finding the mean (average). Before solving this problem, let’s do a quick review of mean.
Review of Mean

To find the mean, add all of the data and then divide by the number of values in your data set:

**Example:** Find the mean of 12, 11, 4, 5, 15, and 7.

5. $12 + 11 + 4 + 5 + 15 + 7 = 54 \leftarrow \text{Add all of the values.}$

2. $\frac{54}{6} \leftarrow \text{There were 6 values so we divide by 6!}$

\[
\frac{54}{6} = 9, \text{ so the mean is 9.}
\]

Now let’s apply this to finding the average speed (average rate). To find the average speed ($r$), divide the total distance traveled ($d$) by the total time traveled ($t$).

\[
\text{Average Speed (R)} = \frac{\text{Total distance (d)}}{\text{Total time (t)}}
\]

Now we are ready to solve the problem involving average speed:

Darius travels from Sacramento to San Francisco. For the first hour, he drives at a constant speed of 50 miles per hour. Then he increases his speed to 60 miles per hour for the next hour. What was his average speed?

Apply the formula for Average Speed $= \frac{\text{Total distance (d)}}{\text{Total time (t)}}$

There are two different intervals given:

- 1st Interval: 1 hour @ 50 mph: Distance = 50 miles
- 2nd Interval: 1 hour @ 60 mph Distance = 60 miles

\[
\text{Total Distance} = \frac{\text{1st Distance} + \text{2nd Distance}}{\text{1st Time} + \text{2nd Time}} = \frac{50 + 60}{1 + 1} = \frac{110}{2} = ____
\]

Darius' average speed is ____ mph.
Let’s try a more complicated problem:

**Example:** Miriam drove for 9 hours at 65 miles per hour. She stopped for dinner and then continued her journey, driving the last 6 hours at 70 mph. What was her average speed for the entire trip?

- Distance Traveled in Interval 1: 9 hrs @ 65 mph = 9 \* 65 = ___
- Distance Traveled in Interval 2: 6 hrs @ 70 mph = 6 \* 70 = ___

Total Distance = 1st Distance + 2nd Distance = _______ = _____
Total Time = 1st Time + 2nd Time

Miriam’s average speed was _____ miles per hour.

**On Your Own**

1. Danielle competed in the local marathon. She ran the first ten kilometers in ten minutes, and then slowed her pace to run the last five kilometers in twenty minutes. What was her average speed?

   Total Distance Traveled: ___ + ____ = _____ kilometers
   
   Total Time Traveled: ___ + ___ = ___ minutes
   
   Average Speed = \( \frac{\text{Total Distance}}{\text{Total Time}} \) = ___
   
   Plug in values into formula: \( S = \frac{D}{T} \) = ____
   
   Solve:
   
   On average, she ran 1 kilometer every ___ minutes.
   
   Her rate per minute was ___ km per minute.
2. An airplane traveling from New York to San Francisco flies at a constant speed of 450 miles per hour for the first 3 hours. Over the remaining 3 hours of the trip, the plane increases its speed to 500 miles per hour. Find the average speed of the plane.

- Distance Traveled in Interval 1: ____________
- Distance Traveled in Interval 2: ____________

Solve:
Average Speed = \( \frac{\text{Total Distance}}{\text{Total Time}} \) = _____

The plane's average speed was ____ mph.

3. Rachel is driving from Sacramento to Los Angeles. She drives three hours at a constant speed of 55 miles per hour. However, she then encounters a storm and slows down to 52 miles per hour for the next two hours. In the last hour, she makes up for lost time by increasing her speed to 70 miles per hour. What is Rachel’s average speed?

- Distance Traveled in Interval 1: ______________
- Distance Traveled in Interval 2: ______________
- Distance Traveled in Interval 3: ______________

Solve:
Unit Quiz: The following questions appeared on the CAHSEE.

1. Two pounds of hamburger cost $3.98. At this price, how much will five pounds cost?
   A. $9.95
   B. $9.98
   C. $19.95
   D. $19.98

2. A person drove for 6 hours at an average speed of 45 miles per hour and for 9 hours at an average speed of 55 mph. Find the average speed for the entire trip.
   A. 50 mph
   B. 51 mph
   C. 52 mph
   D. 53 mph

3. Stephanie is reading a 456 page book. During the past 7 days she has read 168 pages. If she continues reading at the same rate, how many more days will it take her to complete the book?
   A. 12
   B. 14
   C. 19
   D. 24

4. Tina is filling a 45 gallon tub at a rate of 1.5 gallons of water per minute. At this rate, how long will it take to fill the tub?
   A. 30.0 minutes
   B. 43.5 minutes
   C. 46.5 minutes
   D. 67.5 minutes
5. An airplane flies 678 miles from Seattle to San Francisco. The trip takes an hour and a half. What is the airplane’s average speed?

A. 402 miles per hour
B. 422 miles per hour
C. 432 miles per hour
D. 452 miles per hour

6. The diameter of a tree trunk varies directly with the age of the tree. A 45 year old tree has a trunk diameter of 18 inches. What is the age of a tree that has a trunk diameter of 20 inches?

A. 47 years
B. 50 years
C. 63 years
D. 90 years

7. Len runs a mile in 8 minutes. At this rate how long will it take him to run a 26-mile marathon?

Which of the following problems can be solved using the same arithmetic operations that are used to solve the problem above?

A. Len runs 26 miles in 220 minutes. How long does it take him to run each mile?

B. A librarian has 356 books to place on 18 shelves. Each shelf will contain the same number of books. How many books can the librarian place on each shelf?

C. A cracker box weighs 200 grams. What is the weight of 100 boxes?

D. Each basket of strawberries weighs 60 grams. How many baskets can be filled from 500 grams of strawberries?
8. A flower shop delivery van traveled these distances during one week: 104.4, 117.8, 92.3, 168.7, and 225.6 miles. How many gallons of gas were used by the delivery van during this week?

What other information is needed in order to solve this problem?
A. The average speed traveled in miles per hour
B. The cost of gasoline per gallon
C. The average number of miles per gallon for the van
D. The number of different deliveries the van made

9. Chris drove 100 kilometers from San Francisco to Santa Cruz in 2 hours and 30 minutes. What computation will give Chris’ average speed, in kilometers per hour?
A. Divide by 2.5
B. Divide by 2.3
C. Multiply 100 by 2.5
D. Multiply 100 by 2.3

10. Before each game, the Harbor High Mudcats sell programs for $1.00 per program. To print the programs, the printer charges $60 plus $0.20 per program. How many programs does the team have to sell to make a profit of $200?
A. 250
B. 300
C. 325
D. 350
Unit VII: Representing Relationships with Graphs

On the CAHSEE, you will be asked to interpret the meanings of graphs and represent information graphically. Graphing is a way to represent data (information) visually. In this unit, we will review the three most common types of graphs and then look at curve graphs. Finally, we will interpret graphs.

There are three basic graph forms:

- Bar Graphs
- Line Graphs
- Circle Graphs (also known as Pie Charts)

For the first two types of graphs, data is organized along two axes:

1. The horizontal axis (or x-axis)
2. The vertical axis (or y-axis)

Example:

Note: Each axis represents a different unit of measure. The point at which the two axes meet (0, 0) is the origin.
A. Bar Graphs

A bar graph is used to show relationships between groups and is an especially effective tool when illustrating large differences between groups.

The bar graph below represents frozen dessert production in February, 2003. The x-axis shows the types of frozen desserts produced and the y-axis shows the quantity of each dessert produced, as measured in millions of gallons.

Source: USDA- NASS (National Agricultural Statistics Service)

Approximately how many gallons of regular ice cream were produced in February? ______________
B. Line Graphs

A line graph shows **progression** and is an effective tool to use when showing **trends**.

The line graph below represents the federal hourly minimum wage since its inception (beginning). As time progresses from October 1938 to September 1997, the hourly minimum wage steadily rises.

**The Federal Hourly Minimum Wage Since Its Inception:**

By how much did the minimum wage increase from October 1939 to March 1956? _____________

A line graph is also useful when **predicting future data**.

If the federal minimum wage had increased from September 1997 to October 1998, which of the following would be the most likely wage in October 1998?

A. $5.00  
B. $5.15  
C. $5.55  
D. $6.75
The line graph below shows the number of widgets produced between 1993 and 1996:

How many widgets were produced in 1995? _______

The line graph below represents the relationship between time (the horizontal axis) and distance (the vertical axis). The straight diagonal line across the graph shows that as one increases, so too does the other. In other words, there is a positive relationship (or correlation) between time and distance.

How many miles are traveled in 5 hours? __________
C. Pie Charts

Pie charts are circular, like a pie. Each section of the pie represents a part, or fraction, of the whole. Pie graphs (often called circle graphs) are used to show how a part of one thing relates to the whole. They are an effective tool for showing percentages.

The following pie chart shows cheese production, by percent, in 2003.

Source: USDA-NASS (National Agricultural Statistics Service)

Which two types of cheese together accounted for about one-half of all production?
D. Non-linear Graphs

Some graphs represent more complex functions (such as quadratic functions). What’s important to learn here is that they do not follow a straight line (as do linear graphs) but, rather, have a curve.

Example: The graph below shows the change in temperature from August, 1999 through August, 2000 in Putnam City, USA.

The graph shows the change in temperature throughout the year. In August, the temperature is very high; as the seasons change, the temperature falls (reaching sub-zero temperatures in the winter months). In spring, the temperature begins to rise again and continues to rise through the summer months.

Can you think of a city or state that has such large fluctuations (ups and downs) in temperature?
Unit Quiz: The following questions appeared on the CAHSEE.

1. Kareem went for a hike. The graph below shows the relationship between his distance and his time.

![Graph showing distance vs. time with a segment marked 'I'](image)

What does the part of the graph marked 'I' represent?

A. A time during which Kareem rested  
B. A time during which the ground was flat  
C. The altitude at which Kareem started his hike  
D. A time which Kareem ran

2. Consider the circle graph shown below.

![Circle graph showing Ramon's 24-hour day](image)

How many hours a day does Ramon spend in school?

A. 2 hours  
B. 4 hours  
C. 6 hours  
D. 8 hours
3. After three hours of travel, Car A is about how many kilometers ahead of Car B?

A. 2  
B. 10  
C. 20  
D. 25

4. The cost of a long distance call charged by each of the two telephone companies is shown on the graph below.

Company A is less expensive than Company B for:

A. all calls  
B. 3 minute calls only  
C. calls less than 3 minutes  
D. calls longer than 3 minutes
5. The graph below shows the time of travel by pupils from home to school. How many pupils must travel for more than 10 minutes?

A. 2
B. 5
C. 7
D. 8
6. John drew a graph of his expenses for March, April, and May. During that time, his electric bill stayed about the same, and his gas bill decreased each month. In May, he had to buy new clothes, increasing his expenses for clothing.

Looking at the graph above, what is most likely the meaning of I, II, and III?

A. I = gas, II = clothing, III = electric
B. I = gas, II = electric, III = clothing
C. I = electric, II = clothing, III = gas
D. I = clothing, II = electric, III = gas
7. The graph below shows the growth in U.S. population from 1860 to 1980.

What was the increase in population from 1870 to 1930?

A. 100,000,000
B. 80,000,000
C. 50,000,000
D. 40,000,000
8. The graph below shows the value of Whistler Company stock at the end of every other year from 1994 to 2000.

From this graph, which of the following was the most probable value of Whistler Company stock at the end of 1992?

A. -$10
B. $1
C. $10
D. $20
9. Best Burger sells cheeseburgers for $1.75 each. Part of a table representing the number of cheeseburgers purchased and their cost is shown below.

<table>
<thead>
<tr>
<th>Number Purchased</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>3.50</td>
</tr>
<tr>
<td>3</td>
<td>5.25</td>
</tr>
<tr>
<td>4</td>
<td>7.00</td>
</tr>
</tbody>
</table>

Which of the following is a portion of the graph of the data in the table?
Unit VIII: Graphing Linear Functions

Graphs can also be used to represent algebraic equations with two variables. These variables are often represented by the letters $x$ and $y$. On the CAHSEE, you will be asked to identify the graph for simple linear functions. Let's begin with a quick review of plotting points on a graph.

A. Plotting Points on a Graph

Points on a graph are identified by the coordinates $(x, y)$.

1. The $x$-coordinate: represents the number of units to the right (+ number) or left (− number) of the origin $(0, 0)$

2. The $y$-coordinate: represents the number of units above (+ number) or below (− number) the origin $(0, 0)$

To plot any point, you must always start at the origin. You have two moves to get to your point on the graph:

1st move is based on the value of the $x$-coordinate: → or ←

2nd move is based on the value of the $y$-coordinate: ↑ or ↓
Example: Plot the point (-2, 1),

In the graph below, we have plotted the point (-2, 1). Remember, the first value is the x-value: -2. Because the value for x is negative, we place it 2 units to the left of the y-axis. Now look at the y-value (the second value): 1. Since this value is positive, we place it 1 unit above the x-axis.

On Your Own

1. Plot the point (-4, 3) on the graph below.
2. Plot the following points:

(-4, -1), (-3, 0), (-2, 1), (-1, 2), (0, 3), (1, 4), (2, 5)

What is the value of $x$ when $y = 0$? _________________

This is known as the **x-intercept** since it is the value for $x$ when it **intercepts** (or crosses) the axis.

Now look at your graph. What is the value of $y$ when $x = 0$? _____

This is known as the **y-intercept** since it is the value for $y$ when it **intercepts** (or crosses) the axis.

(Note: We will learn more about x- and y-intercepts later on.)

Now connect the points. You should have a **straight line** since the points plotted were derived from a **linear equation**: $y = x + 3$. 

B. Identifying Points Using Slope-Intercept Form

Every straight line has an algebraic equation to match it. In this section, we will learn to represent linear equations in graphical form.

**Steps:**

- The first step in graphing linear equations is to make sure that the equation is written in the proper form: \( y = mx + b \)

We call this form the "**slope-intercept form**."

<table>
<thead>
<tr>
<th>Slope-Intercept Form: ( y = mx + b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = \text{slope} = \frac{\text{rise}}{\text{run}} )</td>
</tr>
<tr>
<td>( b = \text{y-intercept} ) (value of ( y ) where ( x = 0 ))</td>
</tr>
</tbody>
</table>

If an equation is written in any other form, we must first convert it to slope-intercept form in order to determine the points for that line. The example below shows how to convert an equation to slope-intercept form:

**Example:** \(-2x + y = 1\)

To convert this equation to the \( y = mx + b \) form, we need to get "\( y \)" by itself on the left side of the equation. Therefore we must add \(+2x\) to both sides of the equation:

\[
\begin{align*}
-2x + y &= 1 \\
+2x + 2x &\quad y = 1 + 2x \\
\end{align*}
\]

Now switch the terms on the right side of the equation so that the variable comes first:

\[ y = 2x + 1 \quad \text{← This is the slope-intercept form.} \]
Now that the equation is in slope-intercept form, the next step is to determine the values for x and y for different points on your line. We do this by creating a chart. Complete the chart below, finding the y-value that corresponds to each x-value. The first one has been done for you.

\[
y = 2x + 1
\]

<table>
<thead>
<tr>
<th>x-value</th>
<th>( y = 2x + 1 )</th>
<th>y-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( y = 2(-3) + 1 )</td>
<td>-5</td>
</tr>
<tr>
<td>-2</td>
<td>( y = 2(-2) + 1 )</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now that you have all of your points, you can **plot** them. When you are finished, **connect** the points. Since this is a linear equation, you should have a **straight line**.
On Your Own: For each equation, complete the chart, plot your points, and draw a line connecting the points.

1. \( y = 2x -1 \)

<table>
<thead>
<tr>
<th>x-value</th>
<th>( y = 2x -1 )</th>
<th>y-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ y = 2x -1 \]
2. \( x + 2 = y \)

<table>
<thead>
<tr>
<th>x-value</th>
<th>( y = ______ )</th>
<th>y-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. \(-x = -y - 1\)

<table>
<thead>
<tr>
<th>x-value</th>
<th>y = ______</th>
<th>y-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
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<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph of the equation \(-x = -y - 1\) on a coordinate plane with x and y values ranging from -5 to 5.](image-url)
C. Examples of Linear Graphs

i. Constant Function

The graph of a **constant function** is a **straight line parallel** to the **x-axis** or **y-axis**.

In the constant function below, for any $x$-value, the $y$-value **remains the same** (constant).

**Example: $y = 3$**

![Graph of y = 3]

Notice that, in the above example, whatever the value of $x$, the $y$-value remains the **same**: (-3, __), (-1, __), (1, __), (3, __)

In the constant function below, for any $y$-value, the $x$-value remains the same (constant).

**Example: $x = 3$**

![Graph of x = 3]

Notice that, in the above example, whatever the value of $y$, the $x$-value remains the **same**: (__ , -3), (__ , -1), (__ , 1), (__ , 3)
ii. Identity Function

In an identity function, the values of $y$ and $x$ are identical. The equation for an identity function is $y = x$.

Example:

There is a direct relationship between $x$ and $y$. As one increases, so too does the other.

$y = x$

Notice that for each point, the values of $x$ and $y$ are the same:
When $x = 3$, $y = __$
When $x = -1$, $y = ___$

iii. Absolute Value Function

An absolute value function is similar to an identity function for all positive values of $y$. However, since the absolute value of a negative number is positive ($|-x| = |x| = x$), any negative value of $x$ in an absolute value function has a positive value for $y$.

$y = |x|$

When $x = -3$, $y = ___$

Notice that when $x$ is negative, $y$ is positive, and when $x$ is positive, $y$ is also positive.
D. Finding the Slope of a Line

In this section, we will learn to identify the slope of a line.

**Slope:** The slope of a line measures the **steepness** of the line. It is represented as a **fraction**.

The phrase “*rise over run*” represents this fraction. **Rise** refers to how many units you move **up or down** from any one point on the graph to the next point (change in the *y*-value). **Run** refers to how many units you move to the **right** from any one point on the graph to the next point (change in the *x*-value):

\[
m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} \quad (\Delta = \text{change})
\]

**Example:** Find the slope of the line below.

\[
slope = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{2}{3} \quad \text{This means up 2 and over 3!}
\]
On Your Own

1. Find the slope of the line below.

\[ \text{Slope} = \underline{\text{ } } \]

2. Find the slope of the line below.

\[ \text{Slope} = \underline{\text{ } } \]
Negative Slope

When a line has a negative slope it moves **down**, from left to right:

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}
\]

This means down ____ and over ____.

In the equation for the above line, the value of \( m \) (the coefficient before the \( x \)- variable) is **negative**.
Finding the Slope from Two Given Points

In this section, we will learn how to find the slope of a line from two points on that line.

**Example:** Find the slope of the straight line that passes through (3, 4) and (4, 6).

We have two points: (3, 4) and (4, 6)

We know that slope = \( \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} \) \( (\Delta = \text{change}) \)

\( \Delta \) in the y-value (rise) is from 4 to 6; this is +2.
\( \Delta \) in the x-value (run) is from 3 to 4; this is +1.

Expressed as a fraction, \( \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2 \) The slope is 2.

**On Your Own**

1. Find the slope of the straight line that passes through points (3, 3) and (4, 4).

2. Find the slope of the straight line that passes through points (2, -1) and (4, 0)
3. Complete the chart below. Remember, if the slope is positive, then m (the coefficient before the x) will be positive. If the slope is negative, m is negative.

<table>
<thead>
<tr>
<th>Sample Equation</th>
<th>Illustration</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = mx + b</td>
<td></td>
</tr>
<tr>
<td>Positive Slope</td>
<td></td>
</tr>
<tr>
<td>Negative Slope</td>
<td></td>
</tr>
<tr>
<td>No Slope</td>
<td></td>
</tr>
<tr>
<td>Undefined Slope</td>
<td></td>
</tr>
</tbody>
</table>
E. Finding the Intercepts

When presented with a linear graph, we can find its equation by determining its slope and its y-intercept. Note that the equation must be in slope-intercept form:

**Slope-Intercept Form:** $y = mx + b$

$m = \text{slope} = \frac{\text{rise}}{\text{run}}$

$b = \text{y-intercept} \ (\text{value of } y \text{ where } x = 0)$

**i. The Y-Intercept**

The y-intercept is the point where the line crosses the y-axis. It is the value of $y$ where $x = 0$. It will cross at the $b$ value of the equation $y = mx + b$. This point is $(0, y)$.

**Example:** Look at the y-intercepts for the two lines below. The top line intersects the y-axis at the point $(0, 3)$. The value of $x$ is 0; it is neither to the right nor to the left of the origin. The value of $y$ is +3; it is 3 units **above** the origin.

Now look at the bottom line. What are the coordinates (the x- and y-values) of the y-intercept? ________________
ii. **The X-Intercept**

The x-intercept is the place where the line crosses the x-axis. It is the value of x where y = 0. This point is (x, 0).

**Example:** Look at the x-intercept in the line below. It intersects the x-axis at the point (4, 0). The value of y is 0; it is neither above or below the origin. The value of x is +4; it is 4 units to the right of the origin.

![Graph showing the x-intercept](image)

**On Your Own**

1. Look at the graph below:

![Graph](image)

What are the coordinates of the x-intercept? ___________

What are the coordinates of the y-intercept? ___________
2. Look at the graph below:

What are the coordinates of the x-intercept? __________
What are the coordinates of the y-intercept? __________

3. Look at the graph below:

What are the coordinates of the x-intercept? __________
What are the coordinates of the y-intercept? __________
iii. Finding the x- and y-intercepts from Equations

We can find the x- or y-intercept of an equation by setting the other coordinate equal to 0.

To find the **y-intercept**, make $x = 0$

**Example:** Find the y-intercept of the equation $y = 2x + 10$

\[
y = 2(0) + 10 \\
y = 0 + 10 \\
y = 10
\]

The **y-intercept** is **10**.
The **coordinates** of the **y-intercept** are **(0, 10)**.

To find the **x-intercept**, make $y = 0$

**Example:** Find the x-intercept of the equation $0 = 2x + 10$

\[
\leftarrow\text{ Solve for } x:
\]

The **x-intercept** is ____
The **coordinates** of the **x-intercept** are (**__, __**).
On Your Own: Find the x- and y-intercepts for each equation.

1. \( y = 2x + 5 \)
   - The coordinates of the y-intercept are 
   - The coordinates of the x-intercept are 

2. \( y = 4x + 1 \)
   - The coordinates of the y-intercept are 
   - The coordinates of the x-intercept are 

3. \( y = 2x + 6 \)
   - The coordinates of the y-intercept are 
   - The coordinates of the x-intercept are 

4. \( y = -3x + 2 \)
   - The coordinates of the y-intercept are 
   - The coordinates of the x-intercept are 

iv. Finding the Equation of a Graph

Now that we know how to find the slope of a line and its y-intercept, we can find the equation of any linear graph: Just find the values of m (the slope) and b (the y-intercept).

Example: Write the equation of the line below in slope-intercept form (y = ___x + ___)

1. The line crosses the y-axis at point (___, ___) so b = ___.

2. Find the value of m. The easiest way to calculate the slope is to look at whole unit changes. Starting from the y-intercept, the next point where we have whole numbers as both x- and y-variables is (___, ___).

Since slope is equal to rise over run, we find the change in the y-values first (rise): the line moves up ___ units (from 1 to ___). Next, look at the change in x-values (run): the line moves to the right ___ unit (from 0 to ___).

$$\frac{\Delta y}{\Delta x} = \text{________}$$

3. Now plug in each value in the slope-intercept form:

$$y = ___x + ___$$
On Your Own

1. Write the equation for the graph below:

   ![Graph with point (1, 2)]

   \[ b = \_\_\_ \quad m = \_\_\_ \quad y = \_\_x + \_\_\_ \]

2. Write the equation for the graph below:

   ![Graph with point (3, 2)]

   \[ b = \_\_\_ \quad m = \_\_\_ \quad y = \_\_x + \_\_\_ \]

3. Two points on a straight line are (0, 2) and (2, 4).

   \[ b = \_\_\_ \quad m = \_\_\_ \quad y = \_\_\_ \]

4. Two points on a straight line are (0, -1) and (1, 0).

   \[ b = \_\_\_ \quad m = \_\_\_ \quad y = \_\_\_ \]
CAHSEE on Target
UC Davis School/University Partnerships
Student Workbook: Algebra & Functions Strand

F. Matching Linear Equations with their Graphs

When matching linear equations with their graphs, there are two important clues to guide us:

- When the slope, \( m \), is positive, the slope is upward (from left to right). When \( m \) is negative, the slope is downward (from left to right).
- The y-intercept, \( b \), shifts the entire line along the y-axis. If \( b \) is positive, the line shifts up from the origin. If \( b \) is negative, the line shifts down.

**Example: \( y = -x + 1 \)**

\( m = -1 \) \hspace{1cm} \text{Downhill slope}

\( b = +1 \) \hspace{1cm} \text{Line shifts up 1 unit from the origin}

Here is the graph for \( y = -x + 1 \):

![Graph of \( y = -x + 1 \)]

**Example: \( y = 2x + 2 \)**

\( m = \_\_\_\_ \) \hspace{1cm} \text{Note: Uphill slope}

\( b = \_\_\_\_ \) \hspace{1cm} \text{Note: Line shifts up 2 units from the origin}

Here is the graph for \( y = 2x + 2 \):

![Graph of \( y = 2x + 2 \)]
On Your Own

Describe the graph for each equation below:

1. \( y = 3x + 5 \)
   \[ m = ____ \quad \text{←– Uphill slope} \]
   \[ b = ____ \quad \text{←– Line shifts up ____ units} \]

2. \( y = -2x + 4 \)
   \[ m = ____ \quad \text{←– ________ slope} \]
   \[ b = ____ \quad \text{←– Line shifts ____ ____ units} \]

3. \( y = \frac{1}{2} + 1 \)
   \[ m = ____ \quad \text{←– ________ slope} \]
   \[ b = ____ \quad \text{←– Line shifts ____ ____ units} \]

4. \( y = -\frac{1}{2} - 1 \)
   \[ m = ____ \quad \text{←– ________ slope} \]
   \[ b = ____ \quad \text{←– Line shifts ____ ____ units} \]

5. \( y = x - 2 \)
   \[ m = ____ \quad \text{←– ________ slope} \]
   \[ b = ____ \quad \text{←– Line shifts ____ ____ units} \]
Unit Quiz: The following problems appeared on the CAHSEE.

1. What is the slope of the line graphed below?

2. What is the slope of the line below?

A. $-\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$
3. The slope of the line shown below is $\frac{2}{3}$. Find the value of $d$?

B. 3  
C. 4  
D. 6  
E. 9

4. What is the slope of the line shown in the graph below?

A. -2  
B. $-\frac{1}{2}$  
C. $\frac{1}{2}$  
D. 2
5. Which of the following equations has a negative slope?

A. \( y = x - 1 \)
B. \( y = x + 1 \)
C. \( y = -x + 1 \)
D. \( -y = -x + 1 \)

6. What is the equation of the graph shown below?

A) \( y = x -1 \)
B) \( y = x + 1 \)
C) \( y = x + 3 \)
D) \( y = x -3 \)
7. Lia used the following process to find the slope of the line described by the equation $3y + 5x = 12$:

Step 1: Subtract 5x from each side. $3y = -5x + 12$

Step 2: Divide each side by 3. $y = -\frac{5}{3} x + 4$

Step 3: The slope of $y = mx + b$ is m. Slope is $\frac{-5}{3}$

According to Lia’s method, what would be the slope of the line described by the equation $ax + by = c$?

A. $-\frac{a}{b}$

B. $\frac{a}{b}$

C. $-\frac{b}{a}$

D. $\frac{b}{a}$
Unit IX: Graphing Non-Linear Functions

On the CAHSEE, you may be asked to identify the correct graph for a non-linear function, such as $y = nx^2$ or $y = nx^3$.

While the graph of a linear function is a straight line, the graph of a non-linear function is a curve.

A. Plotting Points for Non-Linear Graphs

As with linear graphs, charts are helpful in plotting points for non-linear graphs. Choose values for $x$, find the corresponding $y$ values, and plot a smooth curve through the points $(x, y)$.

**Example:** Complete the chart for the function $y = x^2$.

<table>
<thead>
<tr>
<th>Value of $x$</th>
<th>Value of $y$</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$(0)^2 = 0$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$(1)^2 = 1$</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>$(-1)^2 = 1$</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>$(2)^2 = 4$</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td>$(-2)^2 = 4$</td>
</tr>
</tbody>
</table>

Plotting these points, we get the following graph:
On Your Own: Complete the chart for the function \( y = -x^2 \).

<table>
<thead>
<tr>
<th>Value of x</th>
<th>Value of y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Plot and connect your points:

![Graph](image)

How is this different from the graph on the previous page?

________________________________________________________________________

________________________________________________________________________
B. Quadratic Functions and Parabolas

On the CAHSEE, you may be asked to match a quadratic equation with its graph.

**Quadratic** equations are equations in the form \( y = ax^2 \)

**Note:** They can also be in the form \( y = ax^2 + c \) or ...

\[ y = ax^2 + bx + c \]

The graph of a quadratic equation is a **parabola (bowl-shaped)**. It can open **upward** or **downward**:

![Parabola Diagram]

**Example:** \( 2x^2 -3 \)

**Note:** -3 (the \( c \) value) is the **y-intercept**.
C. Cubic Functions

On the CAHSEE, you may be asked to match a cubic function with its graph.

The cubic function is \( y = x^3 \)

All you need to be able to do for the exam is to recognize the graph of a cubic function; there are two basic ones you should be familiar with, and they are easily distinguished from other kinds of graphs

When \( x \) is negative, \( y \) is negative; when \( x \) is positive, \( y \) is positive.

Example: \( y = x^3 \)

Example: \( y = -x^3 \)
Unit Quiz: The following questions appeared on the CAHSEE.

1. Which of the following could be the graph of $y = x^3$?

A. 

B. 

C. 

D.
2. Which of the following is the graph of \( y = \frac{1}{4}x^2 \)?