Introduction to the CAHSEE

The CAHSEE stands for the California High School Exit Exam. The mathematics section of the CAHSEE consists of 80 multiple-choice questions that cover 53 standards across 6 strands. These strands include the following:

- Number Sense (14 Questions)
- Statistics, Data Analysis & Probability (12 Questions)
- Algebra & Functions (17 Questions)
- Measurement & Geometry (17 Questions)
- Mathematical Reasoning (8 Questions)
- Algebra 1 (12 Questions)

What is CAHSEE on Target?

CAHSEE on Target is a tutoring course specifically designed for the California High School Exit Exam (CAHSEE). The goal of the program is to pinpoint each student’s areas of weakness and to then address those weaknesses through classroom and small group instruction, concentrated review, computer tutorials and challenging games.

Each student will receive a separate workbook for each strand and will use these workbooks during their tutoring sessions. These workbooks will present and explain each concept covered on the CAHSEE, and introduce new or alternative approaches to solving math problems.

What is Algebra?

Algebra is a branch of mathematics that substitutes letters (called variables) for unknown numbers. An algebraic equation equates two expressions. It can be thought of as a scale that must remain in balance. Whatever is done to one side of the equation (e.g., addition, subtraction, multiplication, division) must be done to the other side. In this workbook, each unit will introduce and teach a new concept in algebra.
Unit 1: Foundational Concepts for Algebra

On the CAHSEE, you will be asked to solve linear equations and inequalities in one variable. The key to solving any equation or inequality is to get the unknown variable alone on one side of the equation.

Example: \(2x + 5 = 11\)

\[
\begin{align*}
2x &= 6 \\
x &= 3
\end{align*}
\]

\(\text{Variable Isolated}\)

Equations must balance at all times. Therefore, whatever you do to one side of the equation, you must do to the other side as well.

Example: \(2x + 5 = 11\)

\[
\begin{align*}
2x + 5 - 5 &= 11 - 5 \\
2x &= 6 \\
\frac{2x}{2} &= \frac{6}{2} \\
x &= 3
\end{align*}
\]

\(\text{Variable Isolated}\)

There are three important concepts that come into play when solving algebra equations:

- Adding opposites to get a sum of 0
- Multiplying reciprocals to get a product of 1
- Using the distributive property

We will look at each of these individually and examine how they are used in algebra.
A. Taking the Opposite

**Rule:** To take the **opposite** of the number, simply **change the sign**.

The number line below shows the relationship between opposites. The opposites of the positive integers are their corresponding negative integers; the opposites of the negative integers are their corresponding positive integers.

![Number Line with Opposites](image)

**Example:** The opposite of +7 is -7.

**On Your Own:** Find the opposite of each number.

<table>
<thead>
<tr>
<th>Number</th>
<th>Its Opposite</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>-88</td>
</tr>
<tr>
<td>-88</td>
<td>88</td>
</tr>
<tr>
<td>230</td>
<td>-230</td>
</tr>
<tr>
<td>-230</td>
<td>230</td>
</tr>
<tr>
<td>7903</td>
<td>-7903</td>
</tr>
<tr>
<td>-7903</td>
<td>7903</td>
</tr>
</tbody>
</table>

**Adding Opposites**

**Rule:** The sum of any number and its opposite is always zero.

**Example:** 100 + (-100) = 0
On Your Own: Fill in the blanks to make the following steps true:

1. \(-390 + 390 = 0\)

2. If \(y = -542\), then \(-y = 542\)

3. If \(609 + x = 0\), then \(x = -609\)

4. If \(x = -\frac{1}{5}\), then \(-x = \frac{1}{5}\)

5. \(-32 + 32 = 0\)

6. \(-\frac{1}{5} + \frac{1}{5} = 0\)

7. \(1,340 + (-1,340) = 0\)

8. If \(x - 790 = 0\), what is the value of \(x\)? \(+790\)

9. If \(x + \frac{1}{2} = 0\), what is the value of \(x\)? \(-\frac{1}{2}\)

10. If \(x + 395 = 0\), what is the value of \(x\)? \(-395\)
B. Using the Concept of Opposites in Algebra

The concept of opposites is essential when solving algebraic equations. As we said at the beginning of this unit, the key to solving any equation is to get the unknown variable (for example, $x$) all by itself. Adding opposites (which gives us the sum of 0) is key to isolating an unknown variable. Let's look at the following example:

**Example: $x + 5 = 7$**

In the above problem, we need to find the value of $x$, which is our unknown variable. This means that we need to get rid of the "5" on the left side of the equation so that the $x$ stands alone. Since the sum of any number and its opposite is always zero, we can get rid of the "5" by adding its opposite.

What is the opposite of 5? **-5**

Because equations must balance at all times, we need to add -5 to the other side of the equation as well:

\[
5 + (-5) = 0
\]

\[
x + 5 + (-5) = 7 + (-5)
\]

\[
x = 2
\]

Of course, we are really just subtracting 5 from both sides ($x + 5 - 5 = 7 - 5$), but it is the concept of opposites that is at work here.

Let's look another problem:

**Example: $x - 3 = 9$**

To isolate $x$ on the left side of this new equation, we must take the opposite of -3. The opposite of -3 is +3 (Remember: To take the opposite of any number, just change the sign.) So we must add +3 to both sides of the equation:

\[
-3 + 3 = 0
\]

\[
x - 3 + (3) = 7 + 3
\]

\[
x = 10
\]
On Your Own: Use the concept of opposites to solve for \( x \).

1. \( x + 3 = 13 \)
   \[
   \begin{align*}
   x + 3 - 3 &= 13 - 3 \\
   x &= 10
   \end{align*}
   \]

2. \( -3 + x = 11 \)
   \[
   \begin{align*}
   -3 + 3 + x &= 11 + 3 \\
   x &= 14
   \end{align*}
   \]

3. \( x - 11 = 27 \)
   \[
   \begin{align*}
   x - 11 + 11 &= 27 + 11 \\
   x &= 38
   \end{align*}
   \]

4. \( x - 7 = -15 \)
   \[
   \begin{align*}
   x - 7 + 7 &= -15 + 7 \\
   x &= -8
   \end{align*}
   \]

5. \( -3 + x = -24 \)
   \[
   \begin{align*}
   -3 + 3 + x &= -24 + 3 \\
   x &= -21
   \end{align*}
   \]
C. Reciprocals

**Definition:** The reciprocal of a number is its *multiplicative inverse*. When we multiply a number by its reciprocal, we get a *product of 1*.

**Reciprocal** means the *flip-side, or inverse*.

**Example:** The reciprocal of \( \frac{4}{5} \) is \( \frac{5}{4} \).

**Note:** Zero does not have a reciprocal.

**On Your Own:**

1. The reciprocal of \( \frac{7}{3} \) is \( \frac{3}{7} \)

2. The reciprocal of \( \frac{2}{9} \) is \( \frac{9}{2} \)

3. The reciprocal of \( \frac{2}{5} \) is \( \frac{5}{2} \)

4. The reciprocal of \( \frac{3}{8} \) is \( \frac{8}{3} \)

5. The reciprocal of \( \frac{7}{8} \) is \( \frac{8}{7} \)

6. The reciprocal of \( \frac{2}{3} \) is \( \frac{3}{2} \)

7. The reciprocal of \( \frac{4}{9} \) is \( \frac{9}{4} \)
Finding the Reciprocal of a Whole Number

Example: The reciprocal of 6 is \( \frac{1}{6} \) because any whole number can be written as a fraction with a 1 in its denominator: \( 6 = \frac{6}{1} \); we just flip it to get the reciprocal!

The reciprocal of a whole number will always have a numerator of 1!

On Your Own:

1. \( 10 = \frac{10}{1} \)  
   The reciprocal of 10 is \( \frac{1}{10} \)

2. \( 2 = \frac{2}{1} \)  
   The reciprocal of 2 is \( \frac{1}{2} \)

3. \( -5 = \frac{-5}{1} \)  
   The reciprocal of -5 is \( \frac{-1}{5} \)

4. 16  
   The reciprocal of 16 is \( \frac{1}{16} \)

5. 18  
   The reciprocal of 18 is \( \frac{1}{18} \)

6. 21  
   The reciprocal of 21 is \( \frac{1}{21} \)

7. -14  
   The reciprocal of -14 is \( \frac{-1}{14} \)

8. -23  
   The reciprocal of -23 is \( \frac{-1}{23} \)
Finding the Reciprocal of a Fraction

Rule: To find the reciprocal of a fraction, flip the numerator and the denominator.

Example: The reciprocal of \(\frac{2}{3}\) is \(\frac{3}{2}\).

Example: The reciprocal of \(5^{-1}\) is \(5^{+1} = 5\).

On Your Own: Find the reciprocal of each number:

\[
\begin{array}{cccc}
\frac{3}{10} & \frac{10}{3} & -\frac{1}{33} & -33 & -1,453 & -\frac{1}{1453} \\
3^{-1} & 3 & x^2 & x^2 & x^2 & \frac{1}{x^2}
\end{array}
\]

Multiplying Reciprocals

Rule: Whenever we multiply two numbers that are reciprocals of one another, the product equals 1.

Example: 12 and \(\frac{1}{12}\) are reciprocals because \(\frac{12}{1} \cdot \frac{1}{12} = \frac{12}{12} = 1\).

Example: \(\frac{1}{5}\) and 5 are reciprocals because \(\frac{5}{1} \cdot \frac{1}{5} = 1\).

On Your Own: Solve each equation.

\[
\begin{align*}
-\frac{1}{9} \cdot -9 &= 1 \\
\frac{3}{4} \cdot \frac{4}{3} &= 1 \\
57 \cdot \frac{1}{57} &= 1 \\
3^{-1} \cdot 3 &= 1 \\
3^2 \cdot \frac{1}{3^2} &= 1 \\
\frac{1}{y} &= 1
\end{align*}
\]
D. Using the Concept of Reciprocals in Algebra

The concept of finding reciprocals is essential in algebra. In order to isolate an unknown variable from its coefficient (the numeric factor of the term), we need to multiply that coefficient by its multiplicative inverse (or reciprocal).

Example:

To isolate $x$, we need to multiply the coefficient by its multiplicative inverse (or reciprocal), which is $\frac{1}{2}$.

Since the equation needs to remain balanced at all times, we also need to multiply the term on the right side of the equation by the same value:

Of course, we are really just dividing both sides by 2, but it is the concept of reciprocals that is at work here.
Let's look at another example:

**Example:** \( \frac{x}{2} = 9 \)

Here, the **coefficient** is \( \frac{1}{2} \) and its **reciprocal** is \( \frac{2}{1} \), or 2. Therefore, to isolate \( x \) and keep the equation in balance, we must multiply both sides by 2:

\[
\frac{x}{2} = 9 \\
2 \left( \frac{x}{2} \right) = 2(9) \quad \text{We multiply both sides by 2}
\]

\[
2 \left( \frac{1}{2} \right) (x) = 18 \quad \text{Notice that} \quad \frac{x}{2} \quad \text{can be written as} \quad \frac{1}{2} \cdot x
\]

\[
1x = 18 \\
x = 18
\]

Now let's look at an example with a negative number:

**Example:** \( -\frac{x}{2} = 9 \)

Here, the **coefficient** is \( -\frac{1}{2} \) and its **reciprocal** is \( \frac{-2}{1} \). Therefore, to isolate \( x \) and keep the equation in balance, we must multiply both sides by \( -2 \):

\[
-\frac{x}{2} = 9 \\
-2 \left( -\frac{x}{2} \right) = -2 \left( 9 \right)
\]

\[
x = -18
\]
On Your Own: Use the concept of reciprocals to solve for $x$.

1. $3x = 21$

$$\frac{3x}{3} = \frac{21}{3}$$

$x = 7$

2. $-4x = 16$

$$\frac{-4x}{-4} = \frac{16}{-4}$$

$x = -4$

3. $\frac{x}{5} = 7$

$$(5)\left(\frac{x}{5}\right) = (5)(7)$$

$x = 35$

4. $-\frac{x}{3} = -8$

$$x = 24$$

5. $-x = 16$

$x = -16$
E. Distributive Property

Example: \(3(4 + 5) = (3 \cdot 4) + (3 \cdot 5) = 12 + 15 = 27\)

According to the distributive property, multiplication may be distributed over addition. In other words, when multiplying a number or variable by two or more terms that are added or subtracted (and closed off in parentheses), the multiplication is distributed to each term. We then add the products.

Note: Following the order of operations, you would get the same result. Solving what is in the parentheses first, you would get the following:

Example: \(3(4 + 5) = 3 \cdot 9 = 27\)

On Your Own: Use the distributive property to find the value of each expression.

1. \(4(5 + 9) = 20 + 36 = 56\)

2. \(3(9 - 1) = 27 - 3 = 24\)

3. \(2(6 - 3) = 12 - 6 = 6\)

4. \(-3(2 - 5) = -6 + 15 = 9\)

5. \(-1(-3 + 2) = 3 - 2 = 1\)
F. Using the Distributive Law in Algebra  2.0

The distributive law is used to simplify algebraic expressions like the following:

**Example:** 3(2x - 4)

In the above expression, the number outside of the parentheses (3) must be multiplied by each term within the parentheses: 2x and -4.

3(2x - 4) = (3 \times 2x) + (3 \times -4) = 6x + (-12) = 6x - 12

**On Your Own:** Use the distributive law to simplify each of the expressions.

1. 2(2x + 5) = 4x + 10
2. -2(-2x - 5) = 4x + 10
3. 4(-x -3) = -4x - 12
4. -3(2x -4) = -6x + 12
5. -8(-3x + 4) = 24x - 32
6. -1(-3x + 14) = 3x - 14

**Note:** The distributive law is also used to solve equations. This will be taught later.

**Note to Tutors:** Concepts and skills from Unit 1 will also appear in the Unit 2 Quiz.
Unit Quiz:

1. Solve for \( x \): \( x + 5 = 24 \) \[ x = 19 \]

2. Solve for \( x \): \( 3x = 54 \) \[ x = 18 \]

3. Simplify the expression \( 2(x + 3) \) \[ 2x + 6 \]

4. Solve for \( x \): \( 3x - 2 = 34 \) \[ x = 12 \]

5. Simplify the expression \( 2(x - 6) \) \[ 2x - 12 \]

6. Solve for \( x \): \(-3(12x) = 36\) \[ x = -1 \]

7. Simplify the expression \(-4(4x + 5)\) \[-16x - 20\]

8. Solve for \( x \): \(-5x = 35\) \[ x = -7 \]
Unit 2: Solving Linear Equations  

Now we are ready to solve a simple equation:

Example: 2x + 3 = 15

To get x all by itself, we must remove everything else on that side of the equation.

\[
2x + 3 = 15 \\
2x + 3 - 3 = 15 - 3 \quad \text{We apply rule of opposites} \\
2x = 12 \quad \text{We combine like terms} \\
\frac{2x}{2} = \frac{12}{2} \quad \text{We apply rule of reciprocals} \\
x = 6
\]

Note: Unlike the order of operations, when isolating an unknown variable, we begin with addition or subtraction, and then move on to multiplication or division.

On Your Own

Example: \( \frac{x}{4} - 2 = 6 \)

\[
\frac{x}{4} - 2 = 6 \\
\frac{x}{4} - 2 + 2 = 6 + 2 \quad \text{Apply rule of opposites} \\
(4) \frac{x}{4} = (4)(8) \quad \text{Apply rule of reciprocals} \\
x = 32
\]
Practice: Solve for $x$.

1. $6x - 3 = 9$
   
   \[
   \begin{align*}
   6x &= 12 \\
   x &= 2
   \end{align*}
   \]

2. $\frac{x}{2} + 5 = 7$
   
   \[
   \begin{align*}
   \frac{x}{2} &= 2 \\
   x &= 4
   \end{align*}
   \]

3. $\frac{x}{3} - 4 = 2$
   
   \[
   \begin{align*}
   \frac{x}{3} &= 6 \\
   x &= 18
   \end{align*}
   \]
Isolating the Unknown Variable 4.0 & 5.0

As mentioned above, whenever we solve an algebra problem, our goal is to get the unknown variable, such as \(x\), all by itself. (Note: We are solving for the unknown variable.) In problems in which the unknown variable appears on both sides of the equation, we need to move one of them to the other side. In other words, we need to get all the \(x\)’s on one side and combine them.

Look at the example below:

\[
2x + 1 = 5x + 10
\]

\[
2x + 1 - 1 = 5x + 10 - 1 \quad \text{We apply rule of opposites on numbers}
\]

\[
2x = 5x + 9 \quad \text{We combine like terms}
\]

\[
2x - 5x = 5x - 5x + 9 \quad \text{We apply rule of opposites on variable}
\]

\[
-3x = 9 \quad \text{We combine like terms}
\]

\[
\frac{-3x}{-3} = \frac{9}{-3} \quad \text{We multiply by reciprocal}
\]

\[
x = -3
\]

**On Your Own:** Solve for \(x\).

1. \(3x + 2 = 2x + 5\)

\[
3x - 2x + 2 = 2x -2x + 5 \quad \text{Apply rule of opposites}
\]

\[
x + 2 = 5 \quad \text{Combine like terms}
\]

\[
x + 2 - 2 = 5 - 2 \quad \text{Apply rule of opposites}
\]

\[
x = 3 \quad \text{Combine like terms}
\]

\[
x = 3
\]
2. \(x - 3 = -4x + 7\)

\[
\begin{align*}
x + 4x - 3 &= -4x + 4x + 7 \\
5x - 3 &= 7 \\
5x - 3 + 3 &= 7 + 3 \\
5x &= 10 \\
\frac{5x}{5} &= \frac{10}{5} \\
x &= 2 \\
\end{align*}
\]

Apply rule of opposites

Combine like terms

Apply rule of opposites

Combine like terms

Divide both sides by 5.

3. \(-2x + 8 = -4x + 10\)

\[
\begin{align*}
-2x + 4x &= 10 - 8 \\
2x &= 2 \\
x &= 1 \\
x &= 1
\end{align*}
\]

4. \(3 - x = x - 3\)

\[
\begin{align*}
-2x &= -6 \\
x &= 3 \\
x &= 3
\end{align*}
\]
Simplifying Linear Equations

Some problems will be slightly more complex than the ones we saw above. For these types of problems, we need to simplify first.

Example: \(3(2x - 5) + 4(x - 2) = 12\)

- Get rid of the parentheses. Use the distributive law:

\[
\begin{align*}
3(2x - 5) + 4(x - 2) &= 12 \\
6x - 15 + 4x - 8 &= 12
\end{align*}
\]

- Combine like terms:

\[
\begin{align*}
6x - 15 + 4x - 8 &= 12 \\
6x + 4x - 15 + 8 &= 12 \\
10x - 7 &= 12
\end{align*}
\]

- Now we have a simple equation to solve:

\[
\begin{align*}
10x - 23 &= 12 \\
10x - 23 + 23 &= 12 + 23 & \text{We add 23 to both sides} \\
10x &= 35 \\
\frac{10x}{10} &= \frac{35}{10} & \text{We multiply both sides by } \frac{1}{10} \text{ (Divide by 10)} \\
x &= \frac{35}{10} & \text{This is an improper fraction: numerator > denominator}
\end{align*}
\]

Change to a mixed number and reduce: \(3\frac{1}{2}\)
Example: 2 \((x + 3) - 3(x + 4) = 15\)

- Use distributive law to get rid of the parentheses.
  
  \[
  2x + 6 - 3x - 12 = 15 
  \]

- Combine like terms:
  
  \[
  -x - 6 = 15 
  \]

- Solve:
  
  \[
  -x - 6 = 15 
  \]
  \[
  -x = 21 
  \]
  \[
  x = -21 
  \]
**Practice:** Simplify the following problems, and then solve.

1. \(5(x - 3) - 3(x - 4) = 5\)

\[
\begin{align*}
5x - 15 - 3x + 12 &= 5 \\
2x - 3 &= 5 \\
2x &= 8 \\
x &= 4
\end{align*}
\]

2. \(2(x + 4) - 3(x + 6) = 1\)

\[
\begin{align*}
2x + 8 - 3x - 18 &= 1 \\
-x -10 &= 1 \\
-x &= 11 \\
x &= -11
\end{align*}
\]

3. \(6(x + 3) + 3(2x -4) = 30\)

\[
\begin{align*}
6x + 18 + 6x -12 &= 30 \\
12x + 6 &= 30 \\
12x &= 24 \\
x &= 2
\end{align*}
\]
**Cross Multiplying**

Some problems will be made up of fractions on either side of the equation. Before we can solve these types of equations, we need to get rid of the fractions. We do this by **cross multiplying**.

Cross multiplying means *multiplying* diagonally *across* the equation sign. The term is shorthand for expressing the mathematical law governing proportions, according to which, "the **product of the means** is **equal to** the **product of the extremes.""

\[
\frac{3}{5} = \frac{6}{10} \quad \rightarrow \quad 3 \cdot 10 = 5 \cdot 6
\]

We can cross multiply to find **an unknown variable** in a proportion problem:

**Example:** Solve for \(x\).

\[
\frac{3}{8} = \frac{9}{x}
\]

\[
3x = 72 \quad \text{When we cross multiply, we get } 3x
\]

\[
x = 24 \quad \text{Now divide both sides by 3 to solve for } x.
\]
Let's look at the following example:

\[
\frac{x + 1}{3} = \frac{3x}{5}
\]

Multiplying diagonally across the equation sign, we get the following:

\[
\frac{x + 1}{3} = \frac{3x}{5}
\]

\[
5(x + 1) = 3(3x)
\]

Here, we multiplied to get rid of the parentheses.

Now we have a regular equation to solve. When solving any equation, the goal is to get the variable, \(x\), all by itself:

\[
5x + 5 = 9x
\]

\[
5 = 4x \quad \text{Subtract 5x from both sides}
\]

\[
\frac{5}{4} = x \quad \text{Divide both sides by 4}
\]

**Note:** If you end up with an improper fraction (numerator > denominator), you need to change it to a mixed fraction:

\[
\frac{5}{4} = 1 \frac{1}{4}
\]

**Note:** On the CAHSEE, the answer might appear as a mixed fraction or an improper fraction.
On Your Own  

**Example:** Simplify the following equation: \( \frac{2x}{3} = \frac{x + 3}{12} \)

- Cross multiply:
  \[
  2x (12) = 3(x + 3)
  \]

- Remove the parentheses:
  \[
  24x = 3x + 9
  \]

- Isolate the \( x \):
  \[
  24x = 3x + 9 \\
  21x = 9 \\
  x = \frac{3}{7}
  \]
Practice:  

1. \( \frac{x}{3} = \frac{2x + 5}{3} \)

\[
x(3) = 3(2x + 5) \\
3x = 6x + 15 \\
-3x = 15 \\
x = -5
\]

2. \( \frac{x + 1}{2} = \frac{x}{4} \)

\[
4(x + 1) = 2x \\
4x + 4 = 2x \\
2x = -4 \\
x = -2
\]

3. \( \frac{3x}{4} = \frac{x - 1}{2} \)

\[
2(3x) = 4(x - 1) \\
6x = 4x - 4 \\
2x = -4 \\
x = -2
\]
Equations Involving Squares and Roots 2.0 & 4.0

On the CAHSEE, you may be asked to solve algebra problems that involve squares and roots.

A square is a number raised to the second power. When we square a number, we multiply it by itself:

**Example:** \(5^2 = 5 \cdot 5 = 25\)

The square root of a number is one of its two equal factors:

**Example:** \(\sqrt{25} = 5\) because 5 is one of the two equal factors of 25.

**Note:** \(\sqrt{25}\) is also -5 because \(-5 \cdot -5\) is 25. However, on the CAHSEE, you will be asked to give only the positive root.

On the CAHSEE, you might get a problem like the following:

Solve for \(x\): \(x^2 = 81\)

In order to isolate \(x\), we must undo the squaring of \(x\): This means perform the inverse operation. The inverse of squaring is taking the square root:

\[
x^2 = 81 \quad \Rightarrow \quad \sqrt{x^2} = \sqrt{81} \quad \Rightarrow \quad x = 9
\]
Here's a slightly more difficult problem:

Solve for \( x \): \( \frac{x}{\sqrt{49}} = -3 \)

For problems involving roots, you should simplify before solving:

\[
\frac{x}{\sqrt{49}} = -3
\]

\[
\frac{x}{7} = -3 \quad \text{Note: We took the square root of 49 (} \sqrt{49} = 7 \text{)}
\]

Now we can solve as we would for any other linear equation:

\[
\frac{x}{7} = -3
\]

\[
\text{What's the next step? Multiply by 7}
\]

\[
x = -21
\]

On Your Own:

Solve for \( x \): \( \sqrt{36} (x) = 18 \)

\[
\sqrt{36} (x) = 18 \quad \text{Simplify before solving.}
\]

\[
\text{Divide both sides by 6 to isolate the } x.
\]

\[
x = 3
\]
Practice (Use the positive roots) 2.0 & 4.0

1. Solve for $x$: \( \frac{x}{\sqrt{64}} = 9 \)
   
   \( x = 72 \)

2. Solve for $x$: \( \sqrt[3]{64} = 8x \)
   
   \( x = \frac{1}{2} \)

3. Solve for $x$: \( x^3 = 27 \)
   
   \( x = 3 \)

4. Solve for $x$: \( \frac{3}{\sqrt{121}} = x \)
   
   \( x = \frac{3}{11} \)

5. Solve for $x$: \( 3 = \frac{x}{\sqrt{144}} \)
   
   \( x = 36 \)

6. Solve for $x$: \( 4x^2 = 100 \)
   
   \( x = 5 \)

7. Solve for $x$: \( \sqrt{169} = 26x \)
   
   \( x = \frac{1}{2} \)

8. \( x^3 = 125 \)
   
   \( x = 5 \)

9. \( x^2 = \frac{1}{16} \)
   
   \( x = \frac{1}{4} \)

10. \( \frac{x}{2} = \sqrt{81} \)
    
    \( x = 18 \)
UNIT QUIZ: The following questions appeared on the CAHSEE.

1. If \( x = -7 \), then \( -x = \) ______
   
   A. \( -7 \)
   
   B. \( \frac{1}{7} \)
   
   C. \( \frac{1}{7} \)
   
   D. \( 7 \)  
   
   Standard 2.0

2. Which of the following is equivalent to \( 4(x + 5) - 3(x + 2) = 14 \)
   
   A. \( 4x + 20 - 3x - 6 = 14 \)
   
   B. \( 4x + 5 - 3x + 6 = 14 \)
   
   C. \( 4x + 5 - 3x + 2 = 14 \)
   
   D. \( 4x + 20 - 3x - 2 = 14 \)  
   
   Standard 4.0

3. Which of the following is equivalent to the equation shown below:

\[
\frac{20}{x} = \frac{4}{x-5}
\]

   A. \( x(x- 5) = 80 \)
   
   B. \( 20 (x - 5) = 4x \)
   
   C. \( 20x = 4(x - 5) \)
   
   D. \( 24 = x + (x - 5) \)  
   
   Standard 4.0

---

1 Questions 1 and 5 are based on concepts taught in Unit 1.
4. Colleen solved the equation $2(2x + 5) = 8$ using the following steps:

<table>
<thead>
<tr>
<th>Given:</th>
<th>$2(2x + 5) = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1:</td>
<td>$4x + 10 = 8$</td>
</tr>
<tr>
<td>Step 2:</td>
<td>$4x = -2$</td>
</tr>
<tr>
<td>Step 3:</td>
<td>$x = -\frac{1}{2}$</td>
</tr>
</tbody>
</table>

To get from Step 2 to Step 3, Colleen –

A. **divided both sides by 4**

B. subtracted 4 from both sides

C. added 4 to both sides

D. multiplied both sides by 4  

**Standard 5.0**

5. The perimeter, $P$, of a square may be found by using the formula

\[
\left(\frac{1}{4}\right)P = \sqrt{A},
\]

where $A$ is the area of the square. What is the perimeter of the square with an area of 36 square inches?

A. 9 inches

B. 12 inches

C. **24 inches**

D. 72 inches  

**Standard 2.0**
An inequality is a relationship between two numbers or expressions in which one number is greater than (>), or less than (<), the other. For the most part, solving linear inequalities is very similar to solving linear equations. Let’s solve two similar problems, first as an equation and secondly as an inequality:

**Equation Example:** Solve the equality: \( x - 7 = -3 \)
\[
\begin{align*}
  x - 7 + 7 &= -3 + 7 \\
  x &= 4
\end{align*}
\]

**Inequality Example:** Solve the inequality: \( x - 7 < -3 \)
\[
\begin{align*}
  x - 7 + 7 &< -3 + 7 \\
  x &< 4
\end{align*}
\]

Let’s look at another example of an inequality:

**Example:** Solve the inequality \( 5x < -10 \)
\[
\begin{align*}
  \frac{5x}{5} &< \frac{-10}{5} \\
  x &< -2
\end{align*}
\]

**Exception:** There is one important way in which inequalities differ from equations: If you multiply by a negative number, you must reverse the inequality sign.

**Example:** Solve the inequality \( \frac{-x}{2} > 7 \)
\[
\begin{align*}
  (-2)\left(\frac{-x}{2}\right) &< (-2) (7) \\
  x &< -14
\end{align*}
\]
**On Your Own:** Solve the following inequalities.  

1. \(6x + (3 - x) > 13\)

   \[
   \begin{align*}
   6x + 3 - x & > 13 \\
   5x & > 10 \\
   x & > 2
   \end{align*}
   \]

2. \(-3x + 5 < 19\)

   \[
   \begin{align*}
   -3x + 5 & < 19 \\
   -3x & < 14 \\
   x & > -\frac{14}{3} = -4\frac{2}{3}
   \end{align*}
   \]

3. \(3x + 5 \geq 4x - 12\)

   \[
   \begin{align*}
   3x + 5 & \geq 4x - 12 \\
   -x & \geq -17 \\
   x & \leq 17
   \end{align*}
   \]
4. \[5c - 4 - 7(c + 3) \leq -c\]

\[
\begin{align*}
5c - 4 - 7c - 21 & \leq -c \\
-2c - 25 & \leq -c \\
-25 & \leq c
\end{align*}
\]

5. \[4(x - 3) < -3(2x + 6)\]

\[
\begin{align*}
4x - 12 & < -6x - 18 \\
10x & < -6 \\
x & < -\frac{3}{5}
\end{align*}
\]

6. \[-2(x - 4) > x + 5\]

\[
\begin{align*}
-2x + 8 & > x + 5 \\
-3x & > -3 \\
x & < 1
\end{align*}
\]
Unit Quiz: The following questions appeared on the CAHSSEE.

1. Which of the following is equivalent to 1 - 2x > 3(x - 2)?
   A. \(1 - 2x > 3x - 2\)
   B. \(1 - 2x > 3x - 5\)
   C. \(1 - 2x > 3x - 6\)
   D. \(1 - 2x > 3x - 7\)

2. Which of the following is equivalent to 9 - 3x > 4(2x - 1)?
   A. \(13 < 11x\)
   B. \(13 > 11x\)
   C. \(10 > 11x\)
   D. \(6x > 0\)

3. Solve for \(x\): \(5(2x - 3) - 6x < 9\)
   A. \(x < -1.5\)
   B. \(x < 1.5\)
   C. \(x < 3\)
   D. \(x < 6\)
Mixed Review: Equations and Inequalities  4.0 & 5.0

1. Solve for \( x \): \[ \frac{3x - 9}{2x} = 15 \]

**Tutors:** You may need to explain to students that they can rewrite this problem as \( \frac{3x - 9}{2x} = \frac{15}{1} \) and then just cross multiply, as they have learned.

\[ x = -\frac{1}{3} \]

2. Solve for \( x \): \[ 6(2x - 7) = 6(4x - 3) \]

\[ x = -2 \]

3. \( 4x - 2 = -2(x - 5) \)

\[ x = 2 \]
4. \[
\frac{3x + 4}{5} = \frac{x - 4}{3}
\]

\[x = -8\]

5. Solve for \(x\): \(3(4x - 3) - 3x < 18\)

\[x < 3\]

6. \[
\frac{x}{3} - 3 > -15
\]

\[x > -36\]

Tutors: For more practice, as well as a more in-depth lesson on solving simple linear equations, refer to the Algebra & Functions workbook.
Unit 4: Equations & Inequalities Involving Absolute Value

On the CAHSEE, you will be asked to solve algebraic equations and inequalities that involve absolute value. Before we look at these kinds of problems, let's do a quick review of absolute value.

A. Review of Absolute Value

**Definition:** The absolute value of a number is its distance from 0. This distance is always expressed as a positive number, regardless of whether the number is positive or negative.

It is easier to understand this by examining a number line:

```
-10  -8  -6  -4  -2  0  2  4  6  8  10
```

| -4 | = 4    | 4 | = 4

Notice that the distance of both numbers from 0 is **4 units**.

**Note:** Distance is never negative!

The absolute value of 4, expressed as |4|, is 4 because it is **4 units from zero**. The absolute value of -4, expressed as |-4|, is also 4 because it is **4 units from zero**.

On the other hand, any number between absolute value bars has **two** possible values: a positive value and a negative value:

**Example:** |3| = 3 or -3

**Note:** While the absolute value is +3, the number between the absolute value bars can be +3 or -3: |3| or |-3|
On Your Own: Find the absolute value.

### Absolute Value

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-345</td>
</tr>
<tr>
<td></td>
<td>-1,002</td>
</tr>
<tr>
<td></td>
<td>1,002</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>-50</td>
</tr>
<tr>
<td></td>
<td>-1/5</td>
</tr>
<tr>
<td></td>
<td>-16</td>
</tr>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>-100</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
</tr>
</tbody>
</table>

If $|x| = 8$, what is $x$? **8 or -8** (Hint: There are 2 answers.)
B. Finding the Absolute Value of an Expression

**Rule:** To find the absolute value of an expression, simplify the expression between the absolute value bars and take the absolute value of the result.

**Example:** \(|18 - 11| = |7| = 7\)

**Example:** \(|11 - 18| = |-7| = 7\)

**On Your Own:** Solve within the absolute value bars; then find the absolute value of the expression (i.e. simplify the expression). The first one has been done for you.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>13 - 15</td>
</tr>
<tr>
<td>(</td>
<td>2 - 16</td>
</tr>
<tr>
<td>(</td>
<td>-18 - 30</td>
</tr>
<tr>
<td>(</td>
<td>-52 + 72</td>
</tr>
<tr>
<td>(14 -</td>
<td>13 -6</td>
</tr>
<tr>
<td>(</td>
<td>-10 + 3</td>
</tr>
<tr>
<td>(</td>
<td>-10 - 3</td>
</tr>
</tbody>
</table>

Now we are ready to solve algebraic equations involving absolute value.
C. Algebraic Equations Involving Absolute Value

**Rule:** To solve an equation involving absolute value, make two separate equations (one positive and one negative) and solve each.

**Example:** \(|x + 2| = 7\)

- **Clear** the absolute value bars and **split** the equation into **two cases** (positive and negative):
  \[
  (x + 2) = 7 \quad \text{and} \quad -(x + 2) = 7 \quad \text{Don’t forget: Everything within the bars will be multiplied by -1.}
  \]

- **Remove** the parentheses in each equation:
  \[
  x + 2 = 7 \quad \text{and} \quad -x - 2 = 7
  \]

- **Solve** each equation:
  \[
  \begin{align*}
  x + 2 &= 7 \\
  x &= 5 \\
  \\
  -x - 2 &= 7 \\
  -x &= 9 \\
  x &= -9
  \end{align*}
  \]

**Answer:** The solution set for \(|x + 2| = 7\) is \{-9, 5\}.

**Note:** Absolute value problems will usually have **two** values for \(x\).

- **Now check** by substituting each answer in the original equation:
  \[
  |5 + 2| = |7| = 7 \quad |\text{-9} + 2| = |-7| = 7
  \]

![Same distance graph](image)
Example: \[ |x - 14| = 9 \]

- **Clear** the absolute value bars and **split** the equation into **two cases** (positive and negative):
  
  \[
  \begin{align*}
  (x - 14) &= 9 \\
  -(x - 14) &= 9
  \end{align*}
  \]

- **Remove** the **parentheses** in each equation:
  
  \[
  \begin{align*}
  x - 14 &= 9 \\
  -x + 14 &= 9
  \end{align*}
  \]

- **Solve** each equation:
  
  \[
  \begin{align*}
  x - 14 &= 9 \\
  x &= 23 \\
  -x + 14 &= 9 \\
  -x &= -5 \\
  x &= 5
  \end{align*}
  \]

  **Answer:** The solution set for \( |x - 14| = 9 \) is \( \{5, 23\} \).

- **Check** by substituting each answer in the original equation:
  
  \[
  \begin{align*}
  |23 - 14| &= 9 \\
  |5 - 14| &= 9 \\
  |-9| &= 9
  \end{align*}
  \]
Practice: Find the solution set for each equation.

1. $|x - 35| = 33$
   \[ x = 2 \quad x = 68 \]

2. $|11 - x| = 35$
   \[ x = -24 \quad x = 46 \]

3. $|x + 10 + x| = 8$
   \[ x = -9 \quad x = -1 \]

4. $|13 - 2x| = 9$
   \[ x = 2 \quad x = 11 \]

5. $|2x + 8| = 14$
   \[ x = 3 \quad x = -11 \]
6. \(|x + 3| - 9 = 12\)

\[
\begin{align*}
x &= 18 \\
x &= -24
\end{align*}
\]

7. \(|x + 3 - 9| = 12\)

\[
\begin{align*}
x &= 18 \\
x &= -6
\end{align*}
\]

8. \(|5 - x + 2x| = 11\)

\[
\begin{align*}
x &= 6 \\
x &= -16
\end{align*}
\]

9. \(|5 - x| + 2x = 11\)

\[
\begin{align*}
x &= 6 \\
x &= \frac{16}{3} = 5 \frac{1}{3}
\end{align*}
\]

10. \(|3 + x + 4| = 11\)

\[
\begin{align*}
x &= 4 \\
x &= -18
\end{align*}
\]

Now we are ready to look at inequalities involving absolute value.
D. Inequalities Involving Absolute Value

There are two different rules for solving inequalities involving absolute value: one for "less than" inequalities and one for "greater than" inequalities.

i. "Less than" and "Less than or Equal to" Inequalities

**Rule:** For problems in form |_______| < #, follow this pattern:

\[-# < \underline{_______} < +#\]

**Example:** Assume \(x\) is an integer and solve for \(x\): \(|2x+3| < 6\)

- **Clear** the absolute value **bars** according to the pattern:

  \[-6 < 2x + 3 < 6\]

- **Isolate the \(x\):**

  \[-6 - 3 < 2x + 3 - 3 < 6 - 3 \quad \leftarrow \text{We apply the rule of opposites}\]
  \[-9 < 2x < 3 \quad \leftarrow \text{We combine like terms}\]
  \[-\frac{9}{2} < x < \frac{3}{2} \quad \leftarrow \text{We divide both sides by 2 to isolate \(x\).}\]

**Answer:** We write the solution to \(|2x+3| < 6\) as \(\{\frac{-9}{2}, \frac{3}{2}\}\).

This means that \(x\) is greater than \(-\frac{9}{2}\) and less than \(\frac{3}{2}\).

We can see this on a number line; the solution set for \(x\) includes everything between the two black circles:

\[\frac{-9}{2} \quad \frac{3}{2}\]

- **Check** your solution. Pick a number between \(-\frac{9}{2}\) and \(\frac{3}{2}\). Plug it into the original equation: \(|2(-\frac{9}{2}) + 3| < 6\)

Does the inequality hold true? **Yes:** \(|-5| < 6 \rightarrow 5 < 6\)
**Integer Only Solutions**

On the CAHSEE, you may be asked to find a solution that involves *integers* only.

**Integers** are positive or negative whole numbers. Fractions and decimals are not integers. (Note: 0 is an integer.)

**Example:** If k is an integer, what is the solution to |x - 2| < 1?

- **Clear** the absolute value *bars* according to the pattern:
  
  \[-1 < x - 2 < 1\]

- **Isolate the x:**
  
  \[-1 < x - 2 < 1\]
  
  \[-1 + 2 < x - 2 + 2 < 1 + 2\]  \[\text{Rule of Opposites}\]
  
  \[1 < x < 3\]  \[\text{We combine like terms}\]

The solution set consists of one integer: \{2\}

- **Now check your solution** in the original problem:
  
  \[|x - 2| < 1?\]
  
  \[|2 - 2| < 1?\]
  
  \[0 < 1\]  \[\sqrt{\text{Yes, it checks out!}}\]
Example: Assume x is an integer & solve for x: \(|3x - 4| < 11\)

- **Clear** the absolute value **bars** according to the pattern:
  
  \[-11 < 3x - 4 < 11\]

- **Isolate** and **solve** for **x**:
  
  \[-\frac{7}{3} < x < 5\]

- Check your solution in the *original* equation by setting x equal to any number within the solution set:

  \[
  \begin{array}{c|c}
  |3x - 4| < 11 & |3x - 4| < 11 \\
  |3(-2) - 4| < 11 & |3(4) - 4| < 11 \\
  |-6 - 4| < 11 & |12 - 4| < 11 \\
  |-10| < 11 & |8| < 11 \\
  10 < 11 & 8 < 11 \\
  \text{True statement} & \text{True statement}
  \end{array}
  \]
**Practice:** Find the solution to each inequality.  

1. \(|x + 5| < 9 \)
   
   \(-14 < x < 4\)

2. \(|5x - 14| < 11\)
   
   \(\frac{3}{5} < x < 5\)

3. \(|2x + 3| < 5\)
   
   \(-4 < x < 1\)  \hspace{1cm} \text{Solution: \{-3, -2, -1, 0\}}

4. \(|5 - 3x| < 13\)
   
   \(6 > x > -2 \frac{2}{3}\)  \hspace{1cm} \text{or} \hspace{1cm} 6 > x > -\frac{8}{3} \)

5. \(|3x + 1| \leq 10\)
   
   \(-\frac{11}{3} \leq x \leq 3\)  \hspace{1cm} \text{or} \hspace{1cm} -3 \frac{2}{3} \leq x \leq 3 \)

Now let's look at "greater than" inequalities:
“Greater than” Inequalities

**Rule:** For problems in form \(|______| > #\), split the inequality into two separate cases:

\(|______| < -# \quad \text{and} \quad |______| > +#

*Flip the inequality sign and change the sign of the #.*

**Example:** Assume \(x\) is an integer and solve for \(x\): \(|2x - 3| > 5\).

- **Split** the inequality into two cases:
  
  \[
  |2x - 3| < -5 \quad |2x - 3| > 5
  \]

- **Clear** the absolute value bars:
  
  \[
  2x - 3 < -5 \quad 2x - 3 > 5
  \]

- **Isolate** and **solve** for the \(x\) in each case:
  
  \[
  \begin{align*}
  2x - 3 + 3 & < -5 + 3 \\
  2x & < -2 \\
  \frac{2x}{2} & < \frac{-2}{2} \\
  x & < -1 \\
  \\
  2x - 3 + 3 & > 5 + 3 \\
  2x & > 8 \\
  \frac{2x}{2} & > \frac{8}{2} \\
  x & > 4
  \end{align*}
  \]

**Answer:** \(x\) is less than -1 or \(x\) is greater than 4:

- **Check** your work.

<table>
<thead>
<tr>
<th>Plug in a number less than -1:</th>
<th>Plug in a number greater than 4:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>2(-2) - 3</td>
</tr>
<tr>
<td>Does it check out? <strong>Yes: 7 &gt; 5</strong></td>
<td>Does it check out? <strong>Yes: 7 &gt; 5</strong></td>
</tr>
</tbody>
</table>
On Your Own

Example: Assume $x$ is an integer and solve for $x$: $|2x + 3| > 5$

- **Split** into 2 inequalities:

  $|2x + 3| < -5$  
  $|2x + 3| > 5$

- **Clear** the absolute value **bars**:

  $2x + 3 < -5$  
  $2x + 3 > 5$

- **Isolate** the $x$ in each case:

  $\frac{2x + 3 - 3}{2} < \frac{-5 - 3}{2}$  
  $\frac{2x + 3 - 3}{2} > \frac{5 - 3}{2}$

  $2x < -8$  
  $2x > 2$

  $\frac{2x}{2} < \frac{-8}{2}$  
  $\frac{2x}{2} > \frac{2}{2}$

  $x < -4$  
  $x > 1$

- **Check** your work by testing different values for $x$ in the original inequality: $|2x + 3| > 5$. Does the inequality hold true? Are your values consistent with the solution?

  For $x < -4$, students can set $x$ equal to -5, -6, -7, and so on. They might also set it equal to -4 and -3 to show that the inequality does not hold for values greater than or equal to -4.

  For $x > 1$, student can set $x$ equal to 2, 3, 4, and so on. They might also set it equal to 1 and 0 to show that the inequality does not hold for values less than or equal to 1.
Example: Assume $x$ is an integer and solve for $x$: $|2x + 4| > 6$.

- **Split** the inequality into **two** cases:

  \[
  |2x + 4| < -6 \quad |2x + 4| > 6
  \]

- **Clear** the absolute value **bars**:

  \[
  2x + 4 < -6 \quad 2x + 4 > 6
  \]

- **Isolate** the $x$ in each case:

  \[
  \begin{align*}
  2x + 4 & < -6 \\
  2x & < -10 \\
  x & < -5 \\
  \\
  2x + 4 & > 6 \\
  2x & > 2 \\
  x & > 1
  \end{align*}
  \]

- **Shade** the number line where the inequality is true:
Practice: Solve each inequality. Remember to follow all steps.  3.0

1. \(|x - 3| > 6.\)

\[
\begin{align*}
\text{x - 3} &< -6 & \text{x - 3} &> 6 \\
x &< -3 & x &> 9
\end{align*}
\]

2. \(|6 + 2x| > 26.\)

\[
\begin{align*}
\text{6 + 2x} &< -26 & \text{6 + 2x} &> 26 \\
2x &< -32 & 2x &> 20 \\
x &< -16 & x &> 10
\end{align*}
\]

3. \(|3x + 11| > 7.\)

\[
\begin{align*}
\text{3x + 11} &< -7 & \text{3x + 11} &> 7 \\
3x &< -18 & 3x &> -4 \\
x &< -6 & x &> -1 \frac{1}{3}
\end{align*}
\]

4. \(|x + 3| > 6\)

\[
\begin{align*}
\text{x + 3} &< -6 & \text{x + 3} &> 6 \\
x &< -9 & x &> 3
\end{align*}
\]
5. $|x - 5| > 3$

\[
\begin{array}{cc}
-5 < -3 & 5 > 3 \\
2 < x & x > 8 \\
\end{array}
\]

6. $|-2x - 1| > 3$

\[
\begin{array}{cc}
-2x - 1 < -3 & -2x - 1 > 3 \\
-2x < -2 & -2x > 4 \\
1 > x & x < -2 \\
\end{array}
\]

For problem 6, shade the number line where the inequality is true:

Now check your solution in the original equation by setting $x$ equal to different numbers within the solution set:

\[
\begin{array}{cc}
|-2x - 1| > 3 & |-2x - 1| > 3 \\
|-2(2) - 1| > 3 & |-2(-3) - 1| > 3 \\
4 - 1 > 3 & |6 - 1| > 3 \\
5 > 3 & |5| > 3 \\
5 > 3 & True statement \\
\end{array}
\]
Unit Quiz: The following questions appeared on the CAHSEE.

1. If $x$ is an integer, which of the following is the solution set for the following inequality: $3|x| = 15$?
   A. $\{0, 5\}$
   B. $\{-5, 5\}$
   C. $\{-5, 0, 5\}$
   D. $\{0, 45\}$

2. Assume $k$ is an integer and solve for $k$.
   
   $10 - 2|k| > 4$

   A. $\{-3, -2, -1, 1, 2, 3\}$
   B. $\{-3, -2, -1, 0, 1, 2\}$
   C. $\{-2, -1, 0, 1, 2\}$
   D. $\{-2, -1, 1, 2, 3\}$

3. If $x$ is an integer, what is the solution to $|x-3| < 1$?

   A. $\{-3\}$
   B. $\{-3, -2, -1, 0, 1\}$
   C. $\{3\}$
   D. $\{-1, 0, 1, 2, 3\}$

4. Assume $y$ is an integer and solve for $y$.

   $|y+2| = 9$

   A. $\{-11, 7\}$
   B. $\{-7, 7\}$
   C. $\{-7, 11\}$
   D. $\{-11, 11\}$
Unit 5: Solving Systems of Equations Algebraically

On the CAHSEE, you will be given a "system of equations." This just means that instead of getting one equation with one unknown variable, such as $2x + 5 = 7$, you will be given two equations and two unknown variables.

**Example of a system of equations:**

\[
\begin{align*}
  x + y &= 15 \\
  x - y &= 1
\end{align*}
\]

**Note:** To solve for 1 variable, we only need 1 equation. However, to solve for 2 variables, we need 2 equations!

There are two methods for solving systems of equations. Let's begin with the first method:

**Method 1: Substitution**

In this method, we begin with one equation and one variable, and solve in terms of the second variable.

**Example:**

\[
\begin{align*}
  x - 2y &= 0 \\
  2x + y &= 15
\end{align*}
\]

**Steps:**

- We need to choose one equation to solve first, and to pick one variable to solve for. We could start with either one, but the first equation is simpler:

  \[
  x - 2y = 0
  \]

We will solve for $x$ in terms of $y$:

\[
\begin{align*}
  x - 2y &= 0 \\
  x - 2y + 2y &= 0 + 2y \quad \text{Add 2y to both sides}.
  \\
  x &= 2y
\end{align*}
\]
We can now plug in the value of $x$, which is $2y$, in the second equation and then solve for $y$:

$$2x + y = 15$$

$$2(2y) + y = 15 \quad \text{Plug in the value of } x: \quad 2y$$

**Note:** Now we have only one variable & can easily solve for it.

$$4y + y = 15 \quad \text{We get rid of the parentheses.}$$

$$5y = 15 \quad \text{We add like terms.}$$

$$\frac{5y}{5} = \frac{15}{5} \quad \text{We divide both sides by 5.}$$

$$y = 3 \quad \text{This is our answer.}$$

Now that we have solved for $y$, we can solve for $x$ by plugging the value of $y$ into the first equation:

$$x - 2y = 0$$

$$x - 2(3) = 0 \quad \text{Plug in the value of } y: \quad 3$$

$$x - 6 = 0$$

$$x = 6$$

We can check our answers by plugging in the values of both $x$ and $y$ into either equation to see if the equation holds true:

$$2x + y = 15$$

$$2(6) + (3) = 15 \quad \text{Plug in values for } x \text{ & } y$$

$$12 + 3 = 15 \quad \text{Do the two sides balance? } \sqrt{Yes}$$
Let's look at another example:

\[
\begin{align*}
2x + 5y &= 11 \\
x + 4y &= 13
\end{align*}
\]

- Let’s solve the **second equation** first since its \(x\) coefficient (the numerical factor that appears before the variable) is 1. *(Note: This is easier to solve than the first equation, whose variable has a coefficient of 2.)*

\[
\begin{align*}
x + 4y &= 13 \\
x + 4y - 4y &= 13 - 4y \quad \text{Subtract 4y from both sides.} \\
x &= 13 - 4y
\end{align*}
\]

- Now that we have a value for \(x\), let’s plug it into the first equation and solve for \(y\):

\[
\begin{align*}
2x + 5y &= 11 \\
2(13 - 4y) + 5y &= 11 \quad \text{Plug in the value of } x: \ 13 - 4y \\
26 - 8y + 5y &= 11 \quad \text{We use distributive law} \\
26 - 3y &= 11 \quad \text{We combine like terms} \\
-3y &= -15 \quad \text{We subtract 26 from both sides} \\
y &= 5 \quad \text{Isolate by dividing both sides by } -3
\end{align*}
\]

- Now that we have a value for \(y\), let’s plug it into the second equation and solve for \(x\).

\[
\begin{align*}
x + 4y &= 13 \\
x + 4(5) &= 13 \quad \text{Plug in the value of } y \\
x + 20 &= 13 \quad \text{Get rid of the parentheses} \\
x &= -7 \quad \text{Subtract 20 from both sides}
\end{align*}
\]

Continued →
Check your work: Plug the \( x \) and \( y \) values into the first equation:

\[
2x + 5y = 11 \\
2(-7) + 5(5) = 11 \\
-14 + 25 = 11
\]

Does this check out? \textbf{Yes}

\textbf{Example:}

\[
7x + 3y = 14 \\
x - 2y = 15
\]

Which equation would you start with? \textbf{Second equation}

Which variable would you solve for first? \( x \)

Explain: \textit{We can get \( x \) by itself by adding 2\( y \) to both sides.}

\textbf{Example:}

\[
3x + y = 21 \\
2x - 3y = 20
\]

Which equation would you start with? \textbf{First equation}

Which variable would you solve for first? \( y \)

Explain: \textit{We can get \( y \) by itself by subtracting 3\( x \) from both sides.}
9.0

Let's look at one last example: \[
\begin{align*}
2x + 4y &= 8 \\
3x + 5y &= 14
\end{align*}
\]

Which equation would you start with here? **Not clear yet.**

In the last two examples, we began by solving for a variable with a **coefficient** of 1. In this example, **neither** equation has a variable with a coefficient of 1.

However, we can convert one of the equations so that we **do** have a coefficient of 1 for the \( x \) variable.

Note: It's easier to work with the first equation because each term can be divided evenly by 2:

\[
\begin{align*}
2x + 4y &= 8 & \text{We can divide each term by 2.} \\
\frac{2x}{2} + \frac{4y}{2} &= \frac{8}{2} \\
x + 2y &= 4
\end{align*}
\]

Now we can proceed as we did with the other examples:

- **Solve for** \( x \):

\[
\begin{align*}
x + 2y &= 4 \\
x + 2y - 2y &= 4 - 2y & \text{We subtract 2y from both sides.} \\
x &= 4 - 2y
\end{align*}
\]

- **Now that we have a value for** \( x \), let's plug it into the second equation and solve for \( y \):

\[
\begin{align*}
3x + 5y &= 14 \\
3(4 - 2y) + 5y &= 14 & \text{Plug in value for } x. \\
12 - 6y + 5y &= 14 & \text{Use distributive law and solve for } y. \\
-y &= 2 \\
y &= -2
\end{align*}
\]
Now that we have a value for \( y \), plug it into the first equation and solve for \( x \).

\[
2x + 4y = 8 \\
2x + 4(-2) = 8 \quad \text{Plug in the value of } y. \\
2x - 8 = 8 \quad \text{Solve for } x. \\
2x = 16 \\
x = \frac{16}{2} = 8
\]

Check your work: Plug the \( x \) and \( y \) values into the second equation:

\[
3(8) + 5(-2) = 14 \\
24 - 10 = 14 \quad \text{✓}
\]

Does this check out? Yes
On Your Own  

Find the solution for the system of equations below:

\[
\begin{align*}
    x + 3y &= 12 \\
    2x + y &= 19
\end{align*}
\]

1. Choose one equation to begin with.

\[x + 3y = 12\]

2. Solve for one variable in terms of the other variable.

\[x = 12 - 3y\]

3. Now substitute the value of the variable in the other equation and solve for your second variable:

\[
\begin{align*}
    2(12 - 3y) + y &= 19 \\
    24 - 6y + y &= 19 \\
    -5y &= -5 \\
    y &= 1
\end{align*}
\]

4. Plug the value of your second variable into the equation you began with, and solve for the first variable:

\[
\begin{align*}
    x + 3(1) &= 12 \\
    x &= 9
\end{align*}
\]

5. Check your work:

\[
\begin{align*}
    2x + y &= 19 \\
    2(9) + (1) &= 19 \\
    18 + 1 &= 19 \checkmark
\end{align*}
\]
Practice: Use the method of substitution to find the solution for the systems of equations below:

1. Solve for $x$ and $y$:

\[
\begin{align*}
3x - 2y &= 14 \\
2x - y &= 0
\end{align*}
\]

$x = -14$ and $y = -28$

2. What is the solution for the system of equations?

\[
\begin{align*}
2x - 6y &= 12 \\
2x + 3y &= 15
\end{align*}
\]

$x = 7$ and $y = \frac{1}{3}$

3. What is the solution for the system of equations?

\[
\begin{align*}
3y - 2x &= 11 \\
y + 2x &= 9
\end{align*}
\]

$x = 2$ and $y = 5$
Method 2: Adding Equations

Sometimes it is easier to eliminate one of the variables by adding the equations. The goal is to create a new equation with only one variable.

Example: What is the solution for the system of equations?

\[
\begin{align*}
4y - 2x &= 10 \\
3y + 2x &= 11
\end{align*}
\]

Notice that the \(-2x\) in the first equation and the \(+2x\) in the second equation give us the sum of 0 because the coefficients in the two terms are opposites. In other words, we can get rid of the \(x\) variable by simply adding the two equations together.

\[
\begin{align*}
4y - 2x &= 10 \\
3y + 2x &= 11 \\
\hline
7y + 0 &= 21
\end{align*}
\]

Now we have one equation with one variable: \(7y = 21\)

\[
7y = 21 \\
y = 3
\]

Now plug the value for \(y\) into either of the two original equations and find the value of \(x\):

\[
\begin{align*}
4y - 2x &= 10 \\
4(3) - 2x &= 10 & \text{We plug in the value of } y. \\
12 - 2x &= 10 & \text{We get rid of the parentheses.} \\
12 - 12 - 2x &= 10 - 12 & \text{We subtract } 12 \text{ from both sides.} \\
-2x &= -2 & \text{Combine like terms.} \\
\frac{-2x}{-2} &= \frac{-2}{-2} & \text{Now divide both sides by } -2. \\
\hline
x &= 1
\end{align*}
\]

Does this check out? \(3(3) + 2(1) = 11 \sqrt{\text{✓}}\)
Let's look at another example:

**Example:** What is the solution for the system of equations?

\[
\begin{align*}
2x + y &= 10 \\
2x + 3y &= 4
\end{align*}
\]

Notice that the \(x\)-coefficients are the same in each equation: 2. Therefore, we cannot eliminate a variable by adding the two equations. However, if we **multiply the second equation by -1**, we can then add the two equations and get a 0 value for the \(x\)-variable.

\[-1 \cdot (2x + 3y = 4) \quad \text{Note: We must multiply each term by -1.}\]

This is equal to \(-2x - 3y = -4\).

Now we can add the two equations:

\[
\begin{align*}
2x + y &= 10 \\
-2x - 3y &= -4 & \text{Line up each term in the equations and add.} \\
0 - 2y &= 6 & \text{Notice that the \(x\)-variables canceled out.}
\end{align*}
\]

Now we have one equation in one variable: \(-2y = 6\)

Let's now solve the equation:

\[
\frac{-2y}{-2} = \frac{6}{-2} \quad \text{Divide both sides by -2.}
\]

\[y = -3\]

Now plug the value for \(y\) into either equation and find the value of \(x\):

\[2x + (-3) = 10 \quad \text{We plug in the value of \(y\).}\]

\[2x - 3 = 10 \quad \text{We get rid of the parentheses.}\]

\[2x - 3 + 3 = 10 + 3 \quad \text{We add 3 to both sides.}\]

\[2x = 13 \quad \text{What should you do now? Divide both sides by 2.}\]

\[\frac{2x}{2} = \frac{13}{2} \quad \text{We divide both sides by 2.}\]

\[x = \frac{13}{2} = 6 \frac{1}{2} \quad \text{We get the value of x.}\]

**Check:** \(2x + y = 10 \rightarrow 2\left(\frac{13}{2}\right) + (-3) = 10 \rightarrow 13 - 3 = 10\)
Rules for Solving Systems of Equations with Addition 9.0

When we add equations, our goal is to eliminate one of the variables so that we can solve one equation with one variable. In order to eliminate a variable, the coefficients (the numbers in front of that variable) must be opposites (e.g. -2 and +2). When we add the two equations, we will get a 0 value for that variable.

As we saw on the previous page, whenever the coefficients are the same, we can multiply each term in one equation by -1. We will then have opposite coefficients and when we add the two equations, we will get a 0 value for that variable.

Exercise: Look at each of the following systems of equations. Write which variable can be eliminated (using addition). If we need to first multiply an equation by -1, indicate this.

\[
\begin{align*}
3x + 4y &= 5 \\
x - 4y &= 10
\end{align*}
\]

The y variable can be eliminated

\[
\begin{align*}
2x - 3y &= 8 \\
-x - 3y &= 12
\end{align*}
\]

First, multiply one equation by -1; then we can add the equations and eliminate the y variable.

\[
\begin{align*}
3x - 3y &= 12 \\
3x + 5y &= 10
\end{align*}
\]

First multiply one equation by -1; then we can add the equations and eliminate the x variable.

\[
\begin{align*}
x + y &= 2 \\
x - 2y &= 8
\end{align*}
\]

Multiply 1 equation by -1 and eliminate x variable

\[
\begin{align*}
2x - y &= 20 \\
3x + y &= 14
\end{align*}
\]

Eliminate y variable by adding the equations.
Let's look at another example:

- We can eliminate the $x$ if we first multiply one equation by $-1$. Then, the coefficients for the $x$-variable in the two equations ($1$ and $-1$) will be **opposites**:

\[
\begin{align*}
  x - 3y &= 15 \\
-1(x + 2y) &= -10
\end{align*}
\]

Now we can add the two equations:

\[
\begin{align*}
  x - 3y &= 15 \\
+ (-x - 2y) &= -10 \\
0x - 5y &= 5
\end{align*}
\]

- Now we can solve for $y$:

\[
\begin{align*}
-5y &= 5 \\
-5y &= 5 \\
y &= -1
\end{align*}
\]

- Plug the value for $y$ into either equation and find the value of $x$:

\[
\begin{align*}
x + 2y &= 10 \\
x + 2(-1) &= 10 \\
x - 2 &= 10 \\
x &= 12
\end{align*}
\]

- Check by plugging in the $x$ and $y$ values into the other equation:

\[
\begin{align*}
x - 3y &= 15 \\
12 - 3(-1) &= 15 \\
12 + 3 &= 15
\end{align*}
\]
9.0

**On Your Own:** Use the method of addition or subtraction to solve the system of equations below:

\[
\begin{align*}
3x + 2y &= 6 \\
4x - 2y &= 15
\end{align*}
\]

- Add or subtract the equations and eliminate a variable:

\[
\begin{align*}
3x + 2y &= 6 \\
+ 4x - 2y &= 15 \\
7x &= 21 \\
x &= 3
\end{align*}
\]

- Solve for the other variable:

\[
\begin{align*}
4(3) - 2y &= 15 \\
12 - 2y &= 15 \\
y &= -\frac{3}{2}
\end{align*}
\]

- Plug in the value of the variable (from step 2) in either equation and solve for the remaining variable:

\[
\begin{align*}
3(3) + 2 \left( -\frac{3}{2} \right) &= 6 \\
9 - 3 &= 6 \checkmark
\end{align*}
\]

- Check your work by plugging the values of both variables in the remaining equation:
**Practice:** Use the addition method to solve:

1. Find the solution to the system of equations.

\[
\begin{align*}
    x - 2y &= 14 \\
    x + 3y &= 9
\end{align*}
\]

\[x = 12 \text{ and } y = -1\]

2. Find the solution to the system of equations.

\[
\begin{align*}
    x - y &= 1 \\
    x + 3y &= 9
\end{align*}
\]

\[x = 3 \text{ and } y = 2\]
Let's examine one last example:

\[
\begin{align*}
4x + 3y &= 12 \\
3x - y &= 9
\end{align*}
\]

Neither term can be eliminated through addition. However, we can create an equivalent equation in which one of the terms can be eliminated.

Look at the second equation: \(3x - y = 9\)

If we **multiply the entire equation by 3**, we get the following:

\[
3(3x - y = 9) = 9x - 3y = 27
\]

We now have two \(y\) variables with opposite coefficients: +3 and -3. We can now add the two equations and eliminate the \(y\) variable:

\[
\begin{align*}
4x + 3y &= 12 \\
+ 9x - 3y &= 27 & \text{Add the equations.} \\
13x + 0y &= 39 \\
13x &= 39
\end{align*}
\]

We can now solve for \(x\):

\[
\begin{align*}
13x &= 39 \\
\frac{13x}{13} &= \frac{39}{13} & \text{Divide both sides by 13.} \\
x &= 3
\end{align*}
\]

We can now plug the \(x\) value into either of the two original equations and solve for \(y\). The second equation is easiest:

\[
\begin{align*}
3(3) - y &= 9 & \text{Plug in the value of } x. \\
9 - y &= 9 & \text{Get rid of parentheses.} \\
y &= 0 & \text{Solve for } y.
\end{align*}
\]

Plug the \(x\) and \(y\) values into the first equation to see if they hold true:

\[
4(3) + 3(0) = 12 \quad \text{Does it check out? Yes}
\]
Practice: Use the method of addition to find the solution to each system of equations:

1. Find the solution to the system of equations.
\[
\begin{align*}
2x + y &= 5 \\
3x - 2y &= 11
\end{align*}
\]
First, we will multiply the first equation by 2:
\[
2(2x + y = 5) \rightarrow 4x + 2y = 10
\]
Now add the two equations:
\[
\begin{align*}
4x + 2y &= 10 \\
+ &\quad 3x - 2y = 11 \\
7x &= 21 \\
x &= 3
\end{align*}
\]
\[
4(3) + 2y = 10 \rightarrow y = -1
\]

2. Find the solution to the system of equations:
\[
\begin{align*}
5x + 3y &= -15 \\
10x + 4y &= 20
\end{align*}
\]
First, we will multiply the first equation by -2:
\[
-2(5x + 3y = -15) \rightarrow -10x - 6y = 30
\]
Now add the two equations:
\[
\begin{align*}
-10x - 6y &= 30 \\
+ &\quad 10x + 4y = 20 \\
-2y &= 50 \\
y &= -25
\end{align*}
\]
\[
10x + 4(-25) = 20 \rightarrow 10x = 120 \rightarrow x = 12
\]
Mixed Practice: Use either method (substitution or addition) to find the solution to each system of equations.

1. Find the solution to the system of equations.
   \[
   \begin{align*}
   x + 2y &= -3 \\
   4x - 4y &= 12
   \end{align*}
   \]
   \[x = 1 \text{ and } y = -2\]

2. Find the solution to the system of equations.
   \[
   \begin{align*}
   x + 2y &= 14 \\
   3x - 2y &= 10
   \end{align*}
   \]
   \[x = 6 \text{ and } y = 4\]

3. Find the solution to the system of equations.
   \[
   \begin{align*}
   3x + y &= 13 \\
   2x + y &= 8
   \end{align*}
   \]
   \[x = 5 \text{ and } y = -2\]

4. Find the solution to the system of equations:
   \[
   \begin{align*}
   3x + 4y &= 12 \\
   2x + 2y &= 4
   \end{align*}
   \]
   \[x = -4 \text{ and } y = 6\]
Unit Quiz: The following problems appeared on the CAHSEE.

1. What is the solution of the system of equations shown below?

\[
\begin{align*}
\text{y} &= 3x - 5 \\
\text{y} &= 2x
\end{align*}
\]

A. (1, -2)  
B. (1, 2)  
C. \((5, 10)\)  
D. (-5, -10)  

Standard 9.0

2. What is the solution of the system of equations shown below?

\[
\begin{align*}
7x + 3y &= -8 \\
-4x - y &= 6
\end{align*}
\]

A. (-2, -2)  
B. \((-2, 2)\)  
C. (2, -2)  
D. (2,2)  

Standard 9.0
3. Which graph represents the system of equations shown below?

\[ y = -x + 3 \]
\[ y = x + 3 \]

Answer: B

Standard: 9.0
On the CAHSEE, you will be given problems in which you must add, subtract, multiply and divide monomial, binomial and polynomial expressions. Let’s begin with a review of the vocabulary.

**A. Vocabulary Review**

**Term:** A term, which is the **basic unit** of an algebraic expression, is a specific **value**. A term can be a **number**:

*Examples:* \(5\), \(\frac{1}{3}\), \(1000\)

A **term** can be the product of a number and one or more **variables**:

*Examples of Terms:* \(4x\) \(\rightarrow\) Product of 4 and \(x\)  
\(\frac{x}{3}\) \(\rightarrow\) Product of \(x\) and \(\frac{1}{3}\)  
\(-5ab\) \(\rightarrow\) Product of \(-5\), \(a\), and \(b\)

**Coefficient:** A coefficient is the **numerical factor** of the term; it is the number that appears before the variable.

**Example:** In the term \(-2xy\), the coefficient is \(-2\):

The above term also includes **two** variables: \(x\) and \(y\).

**Note:** All terms have a numerical coefficient, even if it is not written.

**Example:** \(x^2\) \(\rightarrow\) This term consists of the variable \(x^2\). Notice that there is no number in front of the variable. It is understood that we have \(1x^2\); in other words, the **numerical coefficient is 1**. We do not need to write the coefficient.

What is the coefficient of \(-x^2\)?
**Monomial Expression**: An expression consisting of one term

Examples:

\[4x^2z^2\quad 18abc^3\quad \frac{xy}{3}\quad \frac{3}{ab}\quad x\quad 2\]

Addition or subtraction signs connect separate terms.

**Example**: \(2x + 4\)

1st Term + 2nd Term

---

**Binomial Expression**: An expression consisting of two terms connected by the plus (+) or minus (-) sign.

Examples:

\[2x^2 + 3\quad 4 - x\quad x + \frac{5}{x}\]

\[4x + 3\quad 4x^2 + 3x\quad abc + \frac{3a}{5c}\]

---

**Trinomial Expression**: An expression consisting of three terms connected by the plus (+) or minus (-) sign.

Examples:

\[2x^2 + 3x + 5\quad 4x^2 + x - 2\quad x^2 - 5x - 18\]
**Polynomial Expression:** The prefix *poly* means "many." A polynomial expression is a numerical expression that consists of *many* terms; as with all expressions, each term is connected by a plus (+) or minus (−) sign. The term polynomial is usually used for *four or more* terms.

**Example:** $4x^3 + 2x^2 + 5x + 3$

**Degrees of Polynomials**

Look at the polynomial expression: $x^3 + x^2 + x$

Notice that each term contains a different degree (exponent) of $x$. There are special names for each of these terms:

1. Terms that have an exponent of 1 are called *linear* terms.
   
   **Example:** $5x \quad \text{The exponent (1) doesn't need to be written.}$

2. Terms that have an exponent of 2 are called *quadratic* terms.
   
   **Example:** $2x^2 \quad \text{Only } x \text{ is raised to the 2nd power (not 2)}$

3. Terms that have an exponent of 3 are called *cubic* terms.
   
   **Example:** $-5x^3 \quad \text{Only } x \text{ is raised to the 3rd power (not -5)}$

**Note:** The entire expression is named by the *highest degree* and the *number of terms* in the expression:

**Example:** $x^3 + x^2 + x$ is a *cubic trinomial*
B. Adding & Subtracting Polynomials

Polynomials are expressions that contain multiple terms.

Example: $3x^4 + 2x^3 + 5x^2 + 6x - 8$

The above polynomial expression has five terms.

i. Adding Polynomials

To add polynomials, add like terms. Let's solve a problem together.

Example: $3x^2 + 4x + 3 + 2x^2 + 3x + 2$

• Rewrite the problem so that all like terms are next to one another:

$$3x^2 + 2x^2 + 4x + 3x + 3 + 2$$

Note: Like terms have the same variable and are raised to the same power.

• Now add like terms:

$$5x^2 + 7x + 5$$

Note: To add like terms, we add the coefficients of the like terms. The variables and exponents do not change!

On Your Own:

1. $6x^3 + 2x^2 + 4x + 5 + 8x^3 + 14x^2 + 23x + 18$

$$14x^3 + 16x^2 + 27x + 23$$

2. $12a^3 + 9a^2 + 24a + 34 - 14a^3 + 6a^2 + 12a + 9$

$$-2a^3 + 15a^2 + 36a + 43$$
ii. Subtracting Polynomials

To subtract polynomials, combine like terms.

**Example:** \(3x^2 + 4x + 3 - (2x^2 + 3x + 2)\)

- Get rid of the parentheses. Be sure to **subtract the entire expression**: pay attention to the sign of each term that is subtracted.

\[
3x^2 + 4x + 3 - (2x^2 + 3x + 2) = 3x^2 + 4x + 3 - 2x^2 - 3x - 2
\]

- Rewrite the problem so that all like terms are next to one another:

\[
3x^2 + 4x + 3 - 2x^2 - 3x - 2 = 3x^2 - 2x^2 + 4x - 3x + 3 - 2
\]

- Now combine like terms:

\[
3x^2 - 2x^2 + 4x - 3x + 3 - 2 = x^2 + x + 1
\]

**On Your Own:**

1. \(14x^2 + 9x + 15 - (5x^2 - 6x + 19)\)

\[9x^2 + 15x - 4\]

2. \(11y^2 - 17y + 21 - (-12y^2 + 16y - 13)\)

\[23y^2 - 33y + 34\]
C. Multiplying Monomials by Monomials

In the example below, we must multiply the two monomials in order to simplify the expression.

Example: \((5a^2b^3)(-3ab^3)\)

- Multiply the coefficients (the numbers before the variables).
  
  \[5 \cdot -3 = -15\]

- Multiply similar variables (i.e. variables that have a common base). When multiplying variables that have a common base: add the exponents and keep the base.

Imagine an exponent of 1 here!

- \(a^2 \cdot a = a^{2+1} = a^3\)

- \(b^2 \cdot b^3 = b^{2+3} = b^5\)

- Multiply all terms:
  
  \[-15 \cdot a^3 \cdot b^5 = -15a^3b^5\]

On Your Own:

\((-3x^2)(8x^3) = \; -3 \cdot 8 \; \cdot \; x^{2+3} = -24x^5\)

\((-2x^2)(-7y^2) = \; (-2)(-7) \; \cdot \; x^2y^2 = 14x^2y^2\)

\(-5a^3b^4 \cdot -8a^2b^5 = (-5)(-8) \; \cdot \; a^{3+2} \cdot b^{4+5} = 40a^5b^9\)

\((3x^2y)(7xy^2) = \; (3)(7) \; \cdot \; x^{2+1} \cdot y^{1+2} = 21x^3y^3\)
D. Dividing Monomials by Monomials

In the example below, we must divide the two monomials in order to simplify the expression.

Example: \( \frac{21x^3y}{24xy^5} \) ← The fraction bar means division!

• Divide the coefficients: Divide out common factors.

\[ \frac{21 \div 3}{24 \div 3} \quad \text{3 is a factor of both 21 and 24 so} \quad \frac{21}{24} = \frac{7}{8} \]

• Divide similar variables (i.e. variables that have a common base). When dividing variables that have a common base, keep the base and subtract the exponent in the denominator from the exponent in the numerator.

\[ \frac{x^3}{x} = x^{3-1} = x^2 \]

\[ \frac{y}{y^5} = y^{1-5} = y^{-4} = \frac{1}{y^4} \]

• Multiply all terms. Remember, when dealing with fractions, we first multiply numerator by numerator; we then multiply denominator by denominator. Your answer should be a simpler fraction than that in the original problem:

\[ \frac{7 \cdot x^2 \cdot 1}{8 \cdot y^4} = \frac{7 \cdot x^2 \cdot 1}{8 \cdot y^4} = \frac{7x^2}{8y^4} \]

Note: No variable appearing in the numerator should appear in the denominator and vise-versa. If the same variable appears in both the numerator and the denominator, you aren't finished solving the problem: It can be simplified further.
On Your Own:  

1. \[ \frac{14x^4y^5}{18x^2y^5} = \frac{14}{18} \cdot x^{4-2} \cdot y^{5-5} = \frac{7}{9}x^2 \]

2. \[ -\frac{12xy^3}{18x^2y} = -\frac{2y^2}{3x} \text{ or } -\frac{2y^2}{3x} \]

3. \[ \frac{9x^5y^4}{18x^4y^3} = \frac{xy}{2} \]

4. \[ \frac{8a^7y^8}{14a^2y^5} = \frac{4a^5y^3}{7} \]

5. \[ \frac{16ab^3}{24a^3b^5} = \frac{2}{3a^2b^2} \]
E. Multiplying Monomials by Binomials

On the CAHSEE, you may be asked to multiply a monomial expression by a binomial expression:

**Example:** $3x(x + 4)$

This expression contains both...

- a monomial $(3x)$ and
- a binomial $(x + 4)$

To multiply monomials by binomials, use the **distributive law**.

**Distributive Law of Multiplication:**
For any three numbers $a$, $b$, and $c$,

$$a (b + c) = (a \times b) + (a \times c) = ab + ac$$

In other words, multiply the monomial **by each term** in the binomial.

**Example:** $3x(x + 4) = (3x \times x) + (3x \times 4) = 3x^2 + 12x$

$\textbf{On Your Own:}$

1. $4x(3x - 8) = 12x^2 - 32x$

2. $3x^2(-2x - 5) = -6x^3 - 15x^2$

3. $-4x(4x^2 + 6x) = -16x^3 - 24x^2$

4. $7x^2 (4x - 11) = 28x^3 - 77x^2$
F. Multiplying Monomials by Trinomials

Just as we used the distributive law to multiply a monomial by a binomial (in the last lesson), we use the distributive law to multiply a monomial by a trinomial.

Example: $3x(x^2 + 2x - 5)$

This expression contains both . . .

- a monomial ($3x$) and
- a trinomial ($x^2 + 2x - 5$)

Apply the distributive law of multiplication:

Example: $3x(x^2 + 2x - 5) = (3x \cdot x^2) + (3x \cdot 2x) + 3x(-5) = 3x^3 + 6x^2 - 15x$

Note: Each term in the answer has different variables:

- $x^3$
- $x^2$
- $x$

Therefore, the terms cannot be combined, and the expression cannot be further simplified.

Example: $-4x^2(3x^2y + 4xy^2 + 5xy) = (-4x^2)(3x^2y) + (-4x^2)(4xy^2) + (-4x^2)(5xy) = -12x^4y - 16x^3y^2 - 20x^3y$

Notice that each term in the answer has different variables:

- $x^4y$
- $x^3y^2$
- $x^3y$

Therefore, the terms cannot be combined (added), and the expression cannot be further simplified.
On Your Own:  10.0

1. \(-2x^2(3x^2 - 14x - 18) = -6x^4 + 28x^3 + 36x^2\)

2. \(3x(-12x^3 - 8x + 20) = -36x^4 - 24x^2 + 60x\)

3. \(5x^2(-10x^2 - 12x + 7) = -50x^4 - 60x^3 + 35x^2\)

4. \(6x(8x^5 + 7x^4 - 6x) = 48x^6 + 42x^5 - 36x^2\)

5. \(-7x^2(-2x^2 - 7x - 8) = 14x^4 + 49x^3 + 56x^2\)

6. \(5xy(x^2y^2 - 6xy - 8x + 9y) = 5x^3y^3 - 30x^2y^2 - 40x^2y + 45xy^2\)

7. \(-8x^2y(-6xy^2 + 7xy - 9x + 8) = 48x^3y^3 - 56x^3y^2 + 72x^3y - 64x^2y\)

8. \(-6x^2(6x^2 - 12y + 7x + 9) = -36x^4 + 72x^3y - 42x^3 - 54x^2\)
G. Multiplying Binomials by Binomials

When multiplying two binomial expressions, use the distributive law to multiply each term in the first expression by each term in the second expression. One way to keep track of all the terms that need to be multiplied is to use the FOIL method.

**FOIL** stands for **First, Outer, Inner, Last**. That is the order in which we multiply the terms.

**Example:** \((3 + 7x)(6 + 2x)\)

Apply the FOIL method:

- Multiply the first term in each expression: \((3 + 7x)(6 + 2x)\)
- Multiply the outer term in each expression: \(3 \cdot 2x = 6x\)
- Multiply the inner term in each expression: \(7x \cdot 6 = 42x\)
- Multiply the last term in each expression: \(7x \cdot 2x = 14x^2\)
- Now **add** all four answers and combine the like terms:

\[
14x^2 + 6x + 42x + 18 = 14x^2 + 48x + 18
\]

**Note:** We end with a trinomial expression. We always write trinomials from **highest degree terms** to **lowest degree terms**.
On Your Own

**Example:** \((x + 2)(x + 3) = \)_____ 

Use the FOIL method to solve the above problem.

\[(x + 2)(x + 3)\]

**First:** \(x^2\)

**Outer:** \(3x\)

**Inner:** \(2x\)

**Last:** \(6\)

Add and combine like terms: \(x^2 + 3x + 2x + 6 = x^2 + 5x + 6\)

**Note:** We can also multiply two binomials by using a **grid**. Look at the example below.

**Example:** \((x + 2)(x + 3)\)

Now add all the terms and combine any like terms: \(x^2 + 5x + 6\)
Example: Simplify \((x - 2)(x - 2) = _____\)

First:
\[x \cdot x = x^2\]

Outer:
\[x \cdot (-2) = -2x\]

Inner:
\[-2 \cdot x = -2x\]

Last:
\[-2 \cdot -2 = 4\]

Now add and combine like terms: \(x^2 - 2x - 2x + 4 = x^2 - 4x + 4\)
On Your Own

Example: \((x - 3)(2x + 4)\)

Use **FOIL**:

- **First**: \(x \cdot 2x = 2x^2\)
- **Outer**: \(x \cdot 4 = 4x\)
- **Inner**: \(-3 \cdot 2x = -6x\)
- **Last**: \(-3 \cdot 4 = -12\)

Combine: \(2x^2 - 2x - 12\)

---

Example: \((x + 8)(x - 8)\)

Use **FOIL**:

- **First**: \(x \cdot x = x^2\)
- **Outer**: \(x \cdot -8 = -8x\)
- **Inner**: \(8 \cdot x = 8x\)
- **Last**: \(8 \cdot -8 = -64\)

Combine: \(x^2 - 64\)
Practice:  

1. $(2x + 5)(3x - 2) = 6x^2 + 11x - 10$

2. $(x + 4)^2$ → **Hint:** Write without exponent: $(x + 4)(x + 4)$  
   
   $x^2 + 8x + 16$

3. $(3y - 3)^2 = 9y^2 - 18y + 9$

4. $(x - 3)^2 = x^2 - 6x + 9$

5. $(x + y)(x - y) = x^2 + y^2$

6. The length of the rectangle is 8 units longer than its width. Find the area of the rectangle.

   $x + 8$

   $x$

   $x^2 + 8x$
H. Dividing Polynomials by Monomials

Polynomials are expressions that contain multiple terms.

To divide a polynomial by a monomial, use the distributive property.

Example: \( \frac{4x^5 + 8x^3 - 16}{2x^2} \)

- Using the distributive property, write every term of the numerator over the monomial term in the denominator.

\[
\frac{4x^5}{2x^2} + \frac{8x^3}{2x^2} - \frac{16}{2x^2}
\]

- Divide every term in the numerator by the monomial term in the denominator.

1st Term: \( \frac{4x^5}{2x^2} = \frac{2x^3}{12x^2} = 2x^3 \)

\[
\frac{4x^5}{2x^2} + \frac{8x^3}{2x^2} - \frac{16}{2x^2}
\]

2nd Term: \( \frac{8x^3}{2x^2} = \frac{4x^1}{12x^2} = 4x \)

- 

3rd Term: \( \frac{16}{2x^2} = \frac{8}{12x^2} = \frac{8}{x^2} \)

Answer: \( 2x^3 + 4x - \frac{8}{x^2} \)
On Your Own: 10.0

1. \[
\frac{14x^3 + 4x^2 + 2}{2x}
\]

\[
7x^2 + 2x + \frac{1}{x}
\]

2. \[
\frac{18a^2b^3 + 36ab^2 + 27ab}{6b}
\]

\[
3a^2b^2 + 6ab + \frac{9a}{2}
\]

3. \[
\frac{12x^4 - 9x^3 + 6x^2}{3x^2}
\]

\[
4x^2 - 3x + 2
\]
I. Multiplying Fraction Monomials

When multiplying monomial expressions involving fractions, multiply numerators by numerators and denominators by denominators, and then solve as a monomial division problem:

Example: \( \frac{9x^2y}{3xy} \cdot \frac{9x^3}{2y^2} \)

- Multiply the numerators together and multiply the denominators together:

\[
\frac{9x^2y}{3xy} \cdot \frac{9x^3}{2y^2} = \frac{9x^2y \cdot 9x^3}{3xy \cdot 2y^2} = \frac{81x^5y}{6xy^3}
\]

- Now you have a simple division problem. First, divide out common factors in the coefficients:

\[
\frac{81}{6} = \frac{27}{2}
\]

- Divide similar variables (Subtract exponents with a common base):

\[
x\text{-variable: } \frac{x^5}{x} = x^{5-1} = x^4
\]

\[
y\text{-variable: } \frac{y}{y^3} = y^{1-3} = y^{-2} = \frac{1}{y^2}
\]

- Answer: \( \frac{27x^4}{2y^2} \)

Note: Your answer should consist of a coefficient, an x-variable and a y-variable. Does it? ________
On Your Own: 10.0

\[
\frac{12a^4b^5}{30a^2b^4} \cdot \frac{4a}{2a^2} = \underline{\quad \quad \quad \quad}\n\]

• Multiply numerators together & multiply denominators together:

\[
\frac{48a^5b^5}{60a^4b^4}
\]

• Divide out common factors in the coefficients:

\[
\frac{48}{60} = \frac{4}{5}
\]

• Divide similar variables (Subtract exponents with a common base):

\[
\frac{a^5}{a^4} = a
\]

\[
\frac{b^5}{b^4} = b
\]

• Answer: \[
\frac{4}{5}ab
\]
J. Dividing Fraction Monomials

Whenever we divide by a fraction, we need to flip, or invert the second fraction and change the sign (from division to multiplication). We then have a multiplication problem.

Example: \( \frac{14ab}{6a^2} \div \frac{2b^3}{3ab^2} \)

- Invert the second fraction & change the operation (from division to multiplication):

\[
\frac{14ab}{6a^2} \cdot \frac{3ab^2}{2b^3}
\]

- Follow the rules for multiplying monomials and reduce fractions.

\[
\frac{42a^2b^3}{12a^2b^3} = \frac{7}{2} = 3 \frac{1}{2}
\]

On Your Own

\[
\frac{3a^4b}{8ab^2} \div \frac{4a^3b}{6a^2b}
\]

- Invert the second fraction and change the operation:

\[
\frac{3a^4b}{8ab^2} \cdot \frac{6a^2b}{4a^3b}
\]

- Follow the rules for multiplying monomials:

\[
\frac{9a^3}{16b}
\]
Mixed Practice:  

1. \(-3a(4a^2 - 13) = -12a^3 + 39a\)

2. \(9m^2 (6m^2 - 5m + 9) = 54m^4 - 45m^3 + 81m^2\)

3. \((4x - 2)^2 = 16x^2 - 16x + 4\)

4. \((6x + 4)(6x - 4) = 36x^2 - 16\)

5. \(\frac{6xy}{3x^2y} \cdot \frac{4x^3}{3y^2} = \frac{24x^4y}{9x^2y^3} = \frac{8x^2}{3y^2}\)

6. \((4x - 8)(2x - 3) = 8x^2 - 28x + 24\)

7. \(\frac{12xy}{8x^2} \div \frac{4x^3}{3xy^2} = \frac{12xy}{8x^2} \cdot \frac{3xy^2}{4x^3} = \frac{36x^2y^3}{32x^5} = \frac{9y^3}{8x^3}\)

8. \(\frac{25a^5b}{40a^3b^6} = \frac{5a^2}{8b^5}\)
9. \((8ab^2)(6a^2b^3) = 48a^3b^5\)

10. \((5a^2b)(-4ab^2) = -20a^3b^3\)

11. \(3x(2x^2 + 5x - 4) - 4x + 3x^2 + 11 = 6x^3 + 18x^2 - 16x + 11\)

12. \(-2xy(6xy + 3y^3) = -12x^2y^2 - 6x^2y^4\)

13. \(-3(-4x^2 + 9x + 15) - x^3 - 3x^2 + 4x = -x^3 + 9x^2 - 23x - 45\)

14. \(\frac{24x^7 + 18x^5 + 15x^2}{3x} = 8x^6 + 6x^4 + 5x\)

15. \(-14x^2 + 12x + 18 - (8x^2 + 20x - 11) = -22x^2 - 8x + 29\)

16. \(15x^3 - 12x^2 + 16x + 13 + 14x^3 + 7x^2 - 9x + 9 = 29x^3 - 5x^2 + 7x + 22\)
Unit Quiz: The following questions appeared on the CAHSEE.

1. Simplify: \(\frac{2x^3 - 8x^2 + 6x}{2x}\)

2. The two rectangles shown below have dimensions as shown. Which of the following expressions represent the area of the shaded region?

A. \(3w + 2\)
B. \(3w - 2\)
C. \(w + 2\)
D. \(w - 2\)

3. Simplify: \((x^2 - 3x + 1) - (x^2 + 2x + 7)\)

A. \(x - 6\)
B. \(-x + 8\)
C. \(-5x - 6\)
D. \(2x^2 - x + 8\)
4. The length of the rectangle below is 6 units longer than the width.

\[
\begin{array}{c}
\phantom{x} \\
x + 6 \\
\phantom{x} \\
x
\end{array}
\]

Which expression could be used to represent the area of the rectangle?

A. \(x^2 + 6x\)  
B. \(x^2 - 36\)  
C. \(x^2 + 6x + 7\)  
D. \(x^2 + 12x + 36\)  

Standard 10.0
Unit 7: Word Problems

On the CAHSEE, you will be given several word problems to solve. In many cases, using algebra will make your job a lot easier.

Example: Larry bought a total of 32 apples and oranges for $52. An apple costs $2, and an orange costs $1. How many apples did he buy?

Note: Even though the problem asks us to solve for only one unknown variable, there are two unknown variables in this problem:
- the number of apples
- the number of oranges

1. The first step is to establish what your two variables represent.
   Let \( x \) = The number of apples Larry bought
   Let \( y \) = The number of oranges Larry bought

2. The second step is to set up your two equations.
   Note: Two variables \( \rightarrow \) Two equations

   \[
   x + y = 32 \quad \text{Total of 32 apples and oranges}
   \]
   \[
   2x + y = 52 \quad x \text{ apples @ $2 and } y \text{ oranges @ $1} = $52
   \]

   Note: We have made a separate algebraic equation for each written sentence in the word problem.

3. The third step is to solve. Earlier, we learned how to solve a system of equations, using either substitution or addition. Which method would you use here? Why? Subtract equations because the two \( y \) variables each have a coefficient of 1. If we subtract one equation from the other, we can eliminate the \( y \)-variable. Note: It makes more sense to subtract the first from the second.
Now that you have chosen your method, solve for both variables:

\[
\begin{align*}
2x + y &= 52 \\
-x + y &= 32 \quad \text{(Subtract to eliminate the y-variable.)} \\
x &= 20 \quad \text{(Solve for } x). \\
2(20) + y &= 52 \quad \text{(Plug } x\text{-value into the first equation)} \\
40 + y &= 52 \quad \text{(Solve for } y). \\
y &= 12
\end{align*}
\]

4. Check your work.

\[
\begin{align*}
x + y &= 32 \quad \text{(Pick an equation to test values.)} \\
20 + 12 &= 32 \quad \text{(Plug in values for } x \text{ and } y. \checkmark)
\end{align*}
\]
Example: Kareem is twice as old as Jamel. Together, their ages add up to 21. How old is Kareem? How old is Jamel?

• Begin by writing what your [two variables represent.]

Let x = Kareem’s age
Let y = Jamel’s age

• The second step is to set up your [two equations.]

x = 2y \quad \text{(Kareem's age) is 2 times as big as y (Jamel's age).}

x + y = 21 \quad \text{their ages add up to 21}

• Solve for both variables:

\frac{3y}{3} = \frac{21}{3} \quad \text{Solve for y.}

y = 7

\text{Now use substitution method to solve for x.}

x + (7) = 21 \quad \text{Substitute: } y = 7

x + 7 - 7 = 21 - 7 \quad \text{Solve for x.}

x = 14

• Check your answers by testing both values in the other equation:

x = 2y

14 = 2(7) \checkmark
Alternative Method: One Equation & One Variable

Let's look at the last problem again:

**Example:** Kareem is twice as old as Jamel. Together, their ages add up to 21. How old is Kareem? How old is Jamel?

We can solve this problem using **one equation** and **one variable**:

- Begin by establishing what your variable represents:
  
  Let \( x \) = Jamel's age

- Represent Kareem's age in terms of Jamel's age (same variable)
  
  Let \( 2x \) = Kareem's age

- Write an equation that expresses the sum of their ages:
  
  \( x + 2x = 21 \)

- Solve for \( x \):
  
  \[
  \begin{align*}
  x + 2x &= 21 \\
  3x &= 21 \\
  x &= 7
  \end{align*}
  \]

  \( x = 7 \) ← ← ← ← This is Jamel's age.

- Now find Kareem's age
  
  \( 2x = 2(7) = 14 \) ← ← ← ← Kareem's age.

**Note:** The big difference between this method and the one used on the previous page is that Kareem is represented by \( 2x \), rather than \( 2y \). This allows us to work with only one variable: \( x \).
On Your Own: 15.0

Solve the following problem using only one variable:

Fred is half as old as Steven. The difference between their ages is 12.

- Represent the ages of Fred and Steven:
  
  Let \( x \) = Fred's age.  
  
  Let \( 2x \) = Stevens' age.  
  
- Write an equation that expresses the difference between their ages:
  
  \[ 2x - x = 12 \]
  
- Solve for \( x \) (Fred's age):
  
  \[
  \begin{align*}
  2x - x &= 12 \\
  x &= 12 \\
  \end{align*}
  
  Fred is 12 years old

- Now find Steven's age:
  
  \[
  \begin{align*}
  x &= 12 \\
  2x &= 2(12) \\
  2x &= 24 \\
  \end{align*}
  
  Steven is 24 years old

- Check your work:

  Is Fred's age half of Steven's age? Yes
  
  Is the difference between their ages equal to 12? Yes
Rate Problems 15.0

To solve rate problems using algebra, there is one more step.

Example: Raoul can cut 120 pizzas in 40 minutes. His friend Mark can cut that many pizzas in one hour. Working together, how many minutes will it take them to cut 120 pizzas?

- Before we can begin to name our variables and set up our equations, we need to determine the rate of work (i.e. pizzas cut per minute):

  **Rate of Work for Raoul:** Raoul cuts 120 pizzas in 40 minutes. This is a rate of 120 : 40, which can be reduced to 3 : 1. Raoul can cut 3 pizzas in one minute.

  **Rate of Work for Mark:** We need to use the same unit of measurement as we did for Mark (minutes). Mark cuts 120 pizzas per 60 minutes. This is a rate of 2 pizzas per minute.

- Since we now know the rates, the unknown is the number of minutes needed for them to cut 120 pizzas together:

  Let $x = \#$ of minutes for Raoul and Mark to cut 120 pizzas.

- Set up the equation: (One variable $\longrightarrow$ One equation)

  $$3x + 2x = 120$$

  **Note:** To find each person's rate, we multiply the number of pizzas cut per minute by the number of minutes worked ($x$).

- Solve:

  $$5x = 120$$
  $$x = 24$$  It will take them 24 minutes to cut 120 pizzas.
CAHSEE on Target
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Answer Key to Student Workbook: Algebra I Strand

Practice 15.0

For each problem, create algebraic equations and explain what each variable represents. Then solve the problem. (Don’t forget to check your work.)

1. Amy has twice as many oranges as Susan. If, together, they have a total of 39 oranges, how many do they each have?

   \[ x + 2x = 39 \]

   Amy has 26 oranges and Susan has 13 oranges.

2. Paul can sort 450 beans in 9 minutes. Eric can sort 360 beans in 9 minutes. If they work together, how long will it take them to sort 999 beans?

   \[ 50x + 40x = 999 \]

   11.1 minutes

3. A farmer sells two types of apples, one for 25 cents a piece and the other for 40 cents a piece. If in one day she sold 100 apples and made $28, how many of each type of apple did she sell?

   \[ x + y = 100 \]
   \[ .25x + .40y = 28 \]

   80 of 1st apple (@ $0.25) and 20 of 2nd (@ $0.40)

4. Rudy is half as old as Jacob. The sum of their ages is 15. How old are Rudy and Jacob?

   \[ x + 2x = 15 \]

   Rudy is 5 years old and Jacob is 10 years old.
Alternatives to Algebra

Look at the following word problem:

Example: I’m thinking of a number. If you multiply my number by 3 and add 2, you get 17. What’s my number?

We can solve this problem using algebra:

With Algebra
1. Translate into an algebraic equation: 3x + 2 = 17
2. Solve for x:
   a) 3x + 2 = 17
   b) 3x = 15
   c) x = 5

We can also solve this problem without algebra:

Without Algebra
We begin at the end, and we undo each operation, from end to beginning. This method is called "working backwards."

1. We begin at the end and so we start at the answer: 17
2. Undo each operation: Do the opposite of what's stated in the problem.
   a) 17 – 2 = 15 \(\text{Subtract 2} \) because it says to "add 2."
   b) \(\frac{15}{3} = 5\) \(\text{Divide by 3} \) because it says to "multiply by 3."

Do you get the same answer as you did using algebra? Yes
On Your Own: Solve each word problem below using both methods.

1. I’m thinking of a number. If you divide my number by 3 and subtract 7, you get 4. What is my number?

   **With Algebra:**
   \[
   \frac{x}{3} - 7 = 4 \\
   \frac{x}{3} = 11 \\
   x = 33
   \]

   **Without Algebra:**
   \[(4 + 7) \times 3 = 33\]

2. Dave gave Charlotte half of his pogs. Charlotte gave Johnnie half of the pogs she received from Dave. Johnnie kept 8 of those pogs and gave the remaining 10 to Dana. How many pogs did Dave have in the beginning?

   **With Algebra**
   \[
   x - \frac{x}{2} - \left(\frac{1}{2}\right)\left(\frac{x}{2}\right) - 8 = 10 \\
   x - \frac{x}{2} - \frac{x}{4} - 8 = 10 \\
   4x - 2x - x - 32 = 40 \\
   x = 72
   \]

   **Without Algebra**
   \[(10 + 8) \times 2 \times 2 = 72\]
3. Five times an unknown number decreased by 7 is 43. Find the number.

**With Algebra**

\[
5x - 7 = 43
\]

\[
5x = 50
\]

\[
x = 10
\]

**Without Algebra**

\[
\frac{(43 + 7)}{5} = 10
\]

4. When 4 is subtracted from half of an unknown number the result is 17. Find the number.

**With Algebra**

\[
\frac{x}{2} - 4 = 17
\]

\[
\frac{x}{2} = 21
\]

\[
x = 42
\]

**Without Algebra**

\[
(17 + 4) \times 2 = 42
\]

Which method do you prefer?  **Answers will vary.**

Do some methods work better for certain problems?  **Yes. Often, there is less computation involved with the "Working Backwards" method.**
Unit Quiz:

1. Mr. Jacobs can correct 150 quizzes in 50 minutes. His student aide can correct 150 quizzes in 75 minutes. Working together, how many minutes will it take them to correct 150 quizzes?

   A. 30
   B. 60
   C. 63
   D. 125

2. Maxine is three years older than Jan. Together, their ages add up to 21. How old is Maxine? How old is Jan

   \[ x + x + 3 = 21 \]

   Maxine is 12 & Jan is 9

3. Max and Ronda have been asked to collate packets for the science fair. If Max can collate 42 packets in 6 minutes and Ronda can collate 56 packets in seven minutes, how long will it take them, working together, to collate 3,075 packets?

   \[ 7x + 8x = 3,075 \]

   205 minutes

4. Ms. Judson bought 9 pizza pies. Some pies were "extra cheese," and the others were plain. She spent a total of $21.00. If each "extra cheese" pizza pie cost $3.00 and each plain pizza pie cost $2.00, how many of each kind of pie did she buy?

   \[ x + y = 9 \]
   \[ 3x + 2y = 21 \]

   3 extra cheese & 6 plain

Tutors: Try out the cooperative learning activities (supplement). They can be used for reinforcement and/or review.

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2 Question 1 appeared on the CAHSEE.
Unit 8: Graphing Linear Equations \(^3\) 6.0, 7.0 & 8.0

On the CAHSEE, you will be asked to identify the graph that matches a linear equation. You will also be asked to compute (figure out) the x- and y-intercepts of an equation and a line. Let's begin with a review of the most important concepts in graphing linear equations:

### A. Using Slope-Intercept Form

**Slope-Intercept Form: \( y = mx + b \)**

- \( m \) represents the **slope**
  \[ \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} \quad (\Delta = \text{change}) \]
- \( b \) represents the **y-intercept**
  \[ \text{value of } y \text{ when } x = 0 \]

**Note:** We use this form when we are given (or can figure out) the **slope** and the **y-intercept**.

If you are given any equation that is not in slope-intercept form, you must **convert it to this form** before plotting points on a graph.

**Example:** \( 3y = -2x + 9 \)

In order to convert this equation to slope-intercept from, we need to isolate \( y \):

\[
\frac{3y}{3} = -\frac{2}{3}x + \frac{9}{3} \rightarrow \quad \text{To isolate } y, \text{ we divide each term by 3.}
\]

\[
y = -\frac{2}{3}x + 3 \rightarrow \quad \text{This is slope-intercept form.}
\]

What is the slope? (What is the value of the coefficient of \( x \)?) \( \frac{-2}{3} \)

What is the y-intercept? (What is the value of \( b \)?) \( 3 \)

We can now plot our points. (See next page.)

---

\(^3\) Tutors: For in-depth lessons on linear graphs, refer back to Algebra & Functions, Unit VIII.
B. Plotting Points

Plot points for the equation \( y = -\frac{2}{3}x + 3 \)

- Begin by plotting the **y-intercept**. The value of the y-intercept is 3. This means that when \( x = 0, y = 3 \). Therefore the point is \((0, 3)\):
• Starting from the y-intercept, plot your next point. To do this, look at the slope of the equation: if the rise is positive, move up; if negative, move down. For the run, always move to the right.

Here, the slope is negative: $-\frac{2}{3}$

Therefore, we move down 2 units. Since the run is always to the right, we move 3 units over to the right:
6.0, 7.0, 8.0

- To plot each succeeding point, we continue to move **down 2 units** and **to the right 3 units**.

![Graph](image)

- Finally, **draw a straight line** to **join** all of your **points**.

*Tutors: Graph should look like this.*
On Your Own  
6.0, 7.0, 8.0

4y + 4 = 12x + 8

Note: The above equation is not in slope-intercept form. Before we plot any points, we must convert the equation to slope-intercept form.

• Put your equation in the slope-intercept form:

    \[ y = 3x + 1 \]

• Plot your first point: the **y-intercept**. The point is \((0, 1)\).

Tutors: Graph in student version is bigger.

• Starting from your y-intercept, plot your second point. Look at the slope of the equation: if the rise is positive, move up; if negative, move down. For the run, always move to the right.

• Plot each succeeding point in the same way.

• Connect the points to form the line. Tutors: Graph should look like one below.
C. Using a Chart

Another way to plot points on a graph is to create a chart.

**Example:** Graph the following function: \( y = 5 - 3x \)

- To find the y-values, substitute each x-value into the equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>14</td>
<td>(-3, 14)</td>
</tr>
<tr>
<td>-2</td>
<td>11</td>
<td>(-2, 11)</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
<td>(-1, 8)</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>(2, -1)</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>(3, -4)</td>
</tr>
</tbody>
</table>

- Plot the \((x, y)\) coordinates and connect your points:
D. Using a Graph

i. Graph to Equation

**Example:** Find the equation for the line below.

If you are given a graph and asked to find the equation, follow these steps:

- Find the value of \( b \). (This is the \( y \)-intercept.) Value of \( b \): \(-4\)

- Find the value of \( m \). (This is the slope.) Remember, the slope is rise over run, so count how many units the line moves **up** (+) or **down** (-) from one point to the next. This is the numerator of the slope. Then, count the number of units that the line moves **to the right** from one point to the next. This will be the denominator.

  \[
  \text{Value of } m: \quad \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2
  \]

- Plug the \( m \) and \( b \) values into the slope-intercept form: 

  \[
  y = mx + b \\
  y = 2x + (-4) = \\
  y = 2x - 4
  \]
ii. Equation to Graph

If you are given an equation and asked to match it with its graph, follow these steps:

- Be sure the equation is in slope-intercept form. If not, convert it to slope-intercept form.

- Identify the values for the slope (m) and the y-intercept (b).

- Look at the graphs and find the one whose slope and intercept are consistent with those of the equation.

**Example:** Given the equation $y = 2x - 6$, we know that the slope is 2 and the y-intercept is -6.

The graph should look like the one below, because the line crosses the y-axis at point (0, -6) and the line moves up 2 units and to the right 1 unit.
On Your Own:  

Which of the following is the graph of \( y = \frac{1}{2}x - 2 \)?

A  

B  

C  

D

Answer: B
E. Verifying Points on a Line

On the CAHSEE, you may be given an equation and asked to verify whether a particular point lies on the line. You may also be given a point and asked to find the equation.

i. Given an equation, verify if a point lies on a line

**Example:** Given the equation, \(8x + 2y = 6\), determine whether the point \((-2, 11)\) lies on the line:

- Convert the equation to slope-intercept form: \(y = mx + b\)

\[
\begin{align*}
2y & = -8x + 6 \\
y & = -4x + 3
\end{align*}
\]

- Plug in your values \((-2, 11)\) and see if the equation \(y = -4x + 3\) holds true.

\[
\begin{align*}
y & = -4x + 3 \\
11 & = -4(-2) + 3 \\
11 & = 11
\end{align*}
\]

Does the point \((-2, 11)\) lie on line? **Yes**

Explain: **The equation is satisfied when \(x = -2\) and \(y = 11\).**

**Note:** We could also solve this problem without first converting the equation to slope-intercept form. Just plug the values for \(x\) and \(y\) into the original equation and see if it holds true:

\[
\begin{align*}
8x + 2y & = 6 \\
8(-2) + 2(11) & = 6 \\
-16 + 22 & = 6 \quad \checkmark
\end{align*}
\]
1. Given the equation, \(2x + y = 4\), determine if the point \((-2, 8)\) lies on the line.  **Yes**

2. Given the equation, \(-3x - 3y = 12\), determine if the point \((6, 2)\) lies on the line.  **No**

3. Does the point \((3, 4)\) lie on the line for the equation \(y = -4x + 5\)?  **No**

4. Given the equation \(3x = 4 + y\), does the point \((4, 8)\) lie on the line?  **Yes**

5. Given the equation \(4y - 8 = 2x\), does the point \((2, -1)\) lie on the line?  **No**

6. Given the equation \(2y = -1 - 2x\), does the point \((-1, \frac{1}{2})\) lie on the line?  **Yes**

7. Does the point \((-4, 3)\) lie on the line for the equation \(y = -3x - 10\)?  **No**

8. Which of the following points lies on the line \(3x + 14y = 15\)?

   A. \((5, 0)\)
   B. \((0, 5)\)
   C. \((3, 14)\)
   D. \((14, 3)\)
ii. Given a point, find the equation

Example: What is the equation of the line that includes the point (4, 3) and has a slope of -3?

There are two different methods to solve this problem:

Method 1: Use the slope-intercept form.

We can determine the equation for any linear line if we have the slope and one point.

• Plug in the \(x\), \(y\) and \(m\) values into our slope-intercept form:

\[
y = mx + b
\]

\[
3 = (-3)(4) + b
\]

• Solve for \(b\):

\[
3 = -12 + b
\]

\[
15 = b
\]

• Now, all we have to do is go back to the slope-intercept form and plug in the values for \(m\) and \(b\):

\[
y = -3x + 15
\]
What is the equation of the line that includes the point (2, 3) and has a slope of -1?

- Plug in the x, y and m values into our slope-intercept form:

\[ y = mx + b \]
\[ 3 = -1(2) + b \]

- Solve for b:

\[ 3 = -1(2) + b \]
\[ 5 = b \]

- Now, all we have to do is go back to the slope-intercept form and plug in the values for m and b:

\[ y = -x + 5 \]
Method 2: Use the point-slope form

Let's look at this problem again:

Example: What is the equation of the line that includes the point \((4, 3)\) and has a slope of \(-3\)?

This time we will solve the problem using the point-slope form.

Point-Slope Form: \(y - y_1 = m(x - x_1)\)

Note: \((x_1, y_1)\) = any point on the line. In this case, we'll substitute it with the point given in the problem: \((4, 3)\)

To use this form, we must be given 1 point and the slope.

Using the point-slope form, we follow these steps:

- Plug in the \(x_1, y_1,\) and \(m\) values into our point-slope form:
  \[
  y - y_1 = m(x - x_1) \\
  y - 3 = -3(x - 4) \\
  y = -3x + 15
  \]

- Isolate the \(y\):
  \[
  y - 3 = -3(x - 4) \\
  y - 3 = -3x + 12 \\
  y = -3x + 15
  \]

- You should now have an equation in slope-intercept form. Write it below:
  \[
  y = -3x + 15
  \]
On Your Own: 7.0

Use the point-slope form to find the equation of a line with the points (3, 2) and a slope of -3.

- Plug in the $x_1$, $y_1$, and $m$ values into our point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -3(x - 3)$$

- Solve for $y$:

$$y - 2 = -3(x - 3)$$
$$y = -3x + 9$$
$$y = -3x + 11$$

- You should now have an equation in slope-intercept form. Write it below:

$$y = -3x + 11$$

Which method (slope-intercept form or point-slope form) do you prefer? Answers will vary.
Practice: Use either the slope-intercept form or the point-slope form to solve the problems below.

1. What is the equation of the line that includes the point (3, 5) and has a slope of -2?
   \[ y = -2x + 11 \]

2. What is the equation of the line that includes the point (6, 7) and has a slope of 3?
   \[ y = 3x - 11 \]

3. What is the equation of the line that includes the point (-3, 2) and has a slope of -4?
   \[ y = -4x - 10 \]

4. What is the equation of the line that includes the point (-3, 9) and has a slope of -3?
   \[ y = -3x \]

5. What is the equation of the line that includes the point (-2, -4) and has a slope of 1?
   \[ y = x - 2 \]

6. What is the equation of the line that includes the point (-1, -1) and has a slope of -1?
   \[ y = -x - 2 \]

7. What is the equation of the line that includes the point (4, 3) and has a slope of \( \frac{1}{4} \)?  
   \[ \text{Answer: } y = \frac{1}{4}x + 2 \]
F. Slopes and Parallel Lines

Key Concepts:

1. **Parallel Lines**: Two lines in a plane are parallel if they never intersect (cross one another).

2. **Slopes of Parallel Lines**: Two lines are parallel if and only if they have the **same slopes** and **different y-intercepts**.

This is all you need to know about parallel lines for the CAHSEE. Make sure you convert all equations to slope-intercept form. Then compare the slopes of each equation. If they are the same, the lines are parallel; if they are different, the lines are not parallel. It’s as simple as that!

**On Your Own:**

Find the slope of a line parallel to \( y = 2x + 5 \) \( m = 2 \)

Find the slope of a line parallel to \( y = -2x + 5 \) \( m = -2 \)

Find the slope of a line parallel to \( 2x + 2y = 6 \) \( m = -1 \)

Find the slope of a line parallel to \( 3x + y = 15 \) \( m = -3 \)

Find the slope of a line parallel to \( y = 3x + 15 \) \( m = 3 \)
1. Are the lines for equations $3x + 4y = 6$ and $6x + 8y = 9$ parallel?
    Yes: Both have slope of $-\frac{3}{4}$

2. We are given two lines. The first line contains the points $(2, 3)$ and $(4, 6)$. The second line contains the points $(0, 0)$ and $(3, 4)$. Are these lines parallel?
    No, 1st line has slope of $\frac{3}{2}$; 2nd line has slope of $\frac{4}{3}$

3. Are the lines for equations $2y + 3 = x$ and $2x + y = 3$ parallel?
    No; in 1st equation, $m = \frac{1}{2}$; in second equation, $m = -2$.

4. Are the lines for equations $x - 3 = y$ and $y - x = 3$ parallel?
    Yes, both have slope of 1.

5. Are the lines for equations $y - x + 4 = 0$ and $x - 4 = y$ parallel?
    No, they are the same line; they have the same intercept.

6. Write an equation of a line that is parallel to that for $y = -2x + 11$.
    Any equation is fine, as long as the value of $m$ is $-2$ and it has a different $y$-intercept.

7. Write an equation of a line that is parallel to that for $y = 2x - 11$.
    Any equation is fine, as long as $m = 2$.  


**Unit Quiz**: The following questions appeared on the CAHSEE.

1. What is the y-intercept of the line $2x - 3y = 12$?
   - A. $(0, -4)$
   - B. $(0, -3)$
   - C. $(2, 0)$
   - D. $(6, 0)$  
   Standard 6.0

2. What are the coordinates of the x-intercept of the line $3x + 4y = 12$?
   - A. $(0, 3)$
   - B. $(3, 0)$
   - C. $(0, 4)$
   - D. $(4, 0)$  
   Standard 6.0

3. What is an equation of the line shown in the graph below?
   - A. $y = \frac{-3}{2}x + 3$
   - B. $y = \frac{-2}{3}x + 2$
   - C. $y = \frac{3}{2}x - 3$
   - D. $y = \frac{2}{3}x - 2$  
   Standard 6.0
4. Which of the following is the graph of \( y = \frac{1}{2}x + 2 \)?

Answer: D  

Standard 6.0
5. If a line passes through the points A and B shown below, approximately where does the line cross the x-axis?

A. between -3 and -2
B. between 0 and -1
C. between 0 and 1
D. between 1 and 2

Standard 6.0
6. What is the equation of the line that includes the point \((9, 3)\) and
has a slope of \(-\frac{4}{3}\)?

\[ y = -\frac{4}{3}x + 15 \]

7. Which of the following points lies on the line of \(4x + 5y = 20\)?

A. \((0, 4)\)
B. \((0, 5)\)
C. \((4, 5)\)
D. \((5, 4)\)

8. Which of the following points lies on the line \(y = x\)?

A. \((-4, -4)\)
B. \((-4, 4)\)
C. \((4, -4)\)
D. \((-4, 0)\)
9. Given the following graph, find the equation.

\[ y = -\frac{3}{2} + 3 \]  

10. Which of the following statements describes parallel lines?
   A. Same y-intercept but different slopes
   B. **Same slope but different y-intercepts**
   C. Opposite slopes but same x-intercepts
   D. Opposite x-intercepts but same y-intercept

11. Which could be the equation of a line parallel to \( y = 4x - 7 \)?
   A. \( y = \frac{1}{4}x - 7 \)
   B. \( y = 4x + 3 \)
   C. \( y = -4x + 3 \)
   D. \( y = -\frac{1}{4}x - 7 \)

12. What is the slope of a line parallel to the line \( y = \frac{1}{3}x + 2 \)?
   Answer: \( \frac{1}{3} \)
13. What is the slope of a line parallel to the line below?

A. $-\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$  

Standard 8.0